Training of secondary school teachers in pre- and post-unity Italy (1810-1920)

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Secondary school mathematics teachers and their training in pre- and post-unity Italy (1810–1920)

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Abstract

This paper traces the evolution of the profession of mathematics teacher in Italy during the transition of the country from a cluster of small, individual states to a single, unified nation. Our focus will be on the plans carried out for teacher training in the period from 1810, when the *Scuola Normale Superiore* in Pisa was created to train prospective teachers, to 1920, when the *Scuole di Magistero* (teacher training schools) were suppressed.

In particular, we have set two objectives:
– to show how the idea changed of what disciplinary and pedagogical knowledge was useful for training future teachers;
– to illustrate the contributions of a significant group of mathematics teachers who worked along several different lines for the improvement of education.

Keywords: mathematics teacher, profession, training, status, local and national laws, textbooks, teacher associations, didactic journals

1 Introduction

The historical and political situation of the period we consider in this paper is characterized by important changes in Italy. Napoleon's domination and the successive Restoration period were followed by the glorious period of the Risorgimento, which led in 1861 to the Unification of Italy. A galaxy of states (Figure 1) with different conditions regarding political status, traditions, administration, economy and school systems merged into the new nation. The desire to hasten the process of cultural unification made the problem of education central for the leaders of the day, whose ranks included many illustrious mathematicians. In particular, secondary schools were assigned a crucial role in the training of the nation's future ruling class. From this point of view, the

problem of teacher training acquired new significance.

Section 2 illustrates the initiatives aimed at teacher training, from the Napoleonic legacy to the creation of a state-controlled, secular educational system, and to the consequent definition from an institutional point of view of the figure of secondary school professor (the Casati Law). It then presents the troubled history of the Teacher Training Schools, created following Unification, with a focus on related legislative aspects, mathematics courses, and debates over methodologies.

Section 3 shows how several outstanding mathematics teachers interpreted the mission and vision of their profession within the new educational system, and illustrates their contributions along three main lines: the production of textbooks, the publication of mathematical journals about didactics, and their involvement in the community of mathematics educators.

Fig. 1 Italy before unification: the dates in the map indicate the years in which the various states were annexed to the new nation (Weech 1945, p. 787)

2 Training of secondary school teachers[1]

2.1 French domination and the creation of the *Scuola Normale Superiore* in Pisa

From the beginning of the nineteenth century up to 1815 Italy was subjected to Napoleonic domination, following which the Italian territory was partly annexed to France and partly divided into satellite states, the two most extensive of them being the *Kingdom of Italy* (roughly Lombardy, Venetia, The Marches) and the *Kingdom of Naples* (the southern part of the Italian peninsula).

As far as instruction is concerned, one of the most important legacies of the Napoleonic domination of Italy was without a doubt the *liceo* (a kind of secondary school with additional more advanced courses): it laid the foundations for education that was state controlled and secular; it gave a central role to the pure and applied sciences in the education of young people; and it affirmed the importance of training citizens who were responsible and aware of their place in society. Mathematics assumed a role of greater importance, analogous to the one that had once been played by the humanities. The quality of teaching was safeguarded by article 55 of the law enacted on September 4, 1802,[2] which required professors at *liceo* and *ginnasio* (a secondary school with a curriculum more limited with respect to *liceo*) to have a university degree. Exceptions were made in cases of those who were especially well-known, or had been qualified to teach at a university, but the government had the final say regarding the rights and qualifications of the proposed teachers. As far as teachers were concerned, there was a slight trend towards professionalization of the category. Although there were teachers of indubitable scientific excellence – less than ten per cent – the majority consisted of literary men, scholars and teachers from the provinces with various kinds of teaching experience behind them; other teachers were professionals such as doctors, architects, lawyers and draughtsmen, who found their way to teaching by various routes. By and large teachers were relatively young, and mostly of local origins (Table 1).

[1] By Livia Giacardi.
[2] The text can be found in Bucci (1976, pp. 253–261).

Table 1. Age of the professors in the 23 *licei* active in 1812 in the Kingdom of Italy (Pagano 2008, p. 467)

Age		
20–29	21	12.80%
30–39	50	30.49%
40–49	49	29.88%
50–59	27	16.46%
60–69	11	6.71%
70–79	6	3.66%
Total	164	100%

The most important initiative aimed at teacher training was the creation in 1810 of the *Scuola Normale Superiore* (SNS) in Pisa, a "branch" of the Paris *École Normale Supérieure*, to train teachers in literature and the sciences for secondary schools. When the school began its activities in November 1813, there was a clear emphasis on the scientific disciplines: the director was the physicist Ranieri Gerbi (1763–1839), the vice-director was mathematician Giovanni Pieraccioli (1782–1843), and the scientific section had the greatest number of students.

Those admitted to the school, which lasted for two years, had to be at least seventeen years old, have completed their studies at the *liceo* and have passed a careful selection process. The *Pensionato Accademico,* a kind of boarding house for students, provided a community life governed by strict rules. In 1813 twenty-four young men were selected.

Students at the SNS had to attend courses at the University of Pisa. To accustom them to exercise critical judgment as well as train them in teaching methodology (Tomasi and Sistoli Paoli 1990, pp. 33–34), they also had to give lectures under the supervision of tutors. Students in the scientific section who aimed at the study of mathematics had to attend the first-year course in differential and integral calculus as well as those in geometry and algebra; in the second year they had to attend courses in mechanics and astronomy (*Notizie storiche* 1871, p. XI).

2.2 From the Restoration to the first years post-Unification

In general, the Restoration period in Italy meant a return to the past for school programs, a greater involvement of ecclesiastical authorities, and strict control over teachers and students, fundamentally dictated by the desire to quell revolutionary spirit. Initiatives and projects for reform were not lacking, but these

were destined to fail because they were not the products of a political and intellectual class that had a genuine interest in carrying them through, something that would happen in pre-Unification Piedmont.

Soon after the fall of Napoleon, the SNS in Pisa was closed, to reopen only in 1846 as a breeding ground for teachers who were capable of disseminating moderate ideas. An ecclesiastic, Ranieri Sbragia, was named rector of the School, and the students were carefully chosen. As one of the most illustrious students, the poet Giosuè Carducci, put it, it felt like a convent. From 1846 up to 1862, when it was renovated by Carlo Matteucci, the SNS produced 57 graduates who were qualified for teaching, of whom only 14 were from the section of physical-mathematical sciences (*Notizie storiche* 1871, pp. XXIV, XXVI). The difficulty of the times, the reactionary politics of Grand Duke Leopold II, the fact that the rector of the SNS was strongly conservative, and the lack of any university chairs, resulted in the impoverishment of the school's cultural life as well. In 1855 the position of tutor of physics and mathematics was held by a very capable scholar, the physicist Riccardo Felici (1819–1902), and among the university classes that students at the SNS had to attend were also those of the mathematical physicist Ottaviano F. Mossotti (1791–1863), an internationally known scientist (Tomasi and Sistoli Paoli 1990, pp. 46–79).

2.2.1 The creation of the Scuole di metodo *in Piedmont*

The first signs of the new climate following the reactionary phase of the Restoration period were seen in the mid-1830s in the Kingdom of Sardinia, where enlightened aristocrats and intellectuals urged King Carlo Alberto towards reform policies. Instruction was seen as a means of training a ruling class capable of modernizing the nation, as well as educating common citizens. After 1836 various articles regarding education appeared in Piedmontese newspapers. The *Subalpino,* Lorenzo Valerio's *Letture Popolari* and the *Gazzetta Piemontese* were especially active in disseminating the ideas of Johann Heinrich Pestalozzi, Raffaello Lambruschini and others who placed an accent above all on the importance of the teaching method. In particular, the writings of Carlo Boncompagni, future Minister of Public Education, and of the educator Vincenzo Troya contributed to raising public awareness of the need to train good teachers in order to have good

schools. These scholars thought of primary teacher training above all, but their work was also important because of the educational principles that inspired them. Troya, for example, proposed a teaching method that, in an epoch in which lessons were usually dictated and imparted in an authoritarian way, appeared almost revolutionary: he maintained the importance of alternating explanatory lectures with more open dialogue (the Socratic method) so as to start with the children's experience to arrive gradually by means of dialogue from the known to the unknown, from the concrete to the abstract, from the simplest ideas to the more complex (Cardinali et al. 1983, pp. 130–136).

On June 4, 1844, King Carlo Alberto officially approved a school for training primary school teachers at the University of Turin.[3] Carlo Alberto called the priest and pedagogue Ferrante Aporti to teach the classes; he was the creator of the first nursery schools in Lombardy-Venetia. Lessons began on August 26, 1844. The numerous students included not only teachers from all over Piedmont, but also secondary school teachers, people of culture, and educators such as Troya, G. Antonio Rayneri, Boncompagni, and others. While there were 170 students officially enrolled, including 57 primary school teachers and 29 secondary school teachers, many more people – as many as 500 (Sideri 1999, p. 362) – came to listen.

The pedagogical assumptions of Aporti were strongly linked to the Christian tradition of education, but were also influenced by the Austrian organization of schools adopted in Lombardy-Venetia. In particular, he maintained the importance, especially in primary schools, of the Socratic method and the intuitive method, and the usefulness of basing teaching on the interests of young people rather than the catechistic and rote method normally used up to then (Sideri 1999, pp. 69–107).

In January 1845 King Carlo Alberto nominated a commission to prepare a plan for more stable, widespread training for teachers and for professors of educational method. The commission's project was the basis for the Royal decrees of 1845

[3] At the end there were written and oral examinations. Passing the examination entitled one to preferential treatment regarding assignation of positions (Capello 1917–1918, pp. 400–401).

(Vidari 1927, pp. 688–689) instituting a *Scuola superiore di metodo* at the University of Turin to qualify the professors of *Metodo* (pedagogy), and calling for the opening of provincial schools for primary teacher training.[4] The professorship in *Metodo* in Turin was assigned to Casimiro Danna; in 1847 it was taken over by Rayneri, with Troya as assistant.

The Boncompagni Law, enacted in 1848 in Piedmont, placed the entire school system under the authority of the Ministry of Public Education, extracting it from the influence of the clergy. Secondary schools, in spite of the introduction of additional courses with more in-depth study of the different subjects, continued to adhere essentially to the old traditional classical model, and little consideration was given to the training of secondary school teachers. The only initiative of interest was determined by the necessity of preparing suitable teachers for upper-level primary teaching in order to fight illiteracy. To this end the Minister of Public Education Pietro Gioia decreed the opening of special schools, both for in-service teachers and for aspiring teachers, in Turin, Genoa, Alessandria, Cuneo and Novara. As far as mathematics was concerned, in the program for the oral admission examinations, only knowledge of arithmetic and the decimal metric system was required. The instructions for the professors and assistants in these schools recommended establishing connections between the various disciplines, and providing not only the notions that the teacher had to know, but also the teaching method. For mathematics it was suggested to propose problems that used notions of physics and the natural sciences, and to teach geometry by beginning with observations; the recommended text was the Italian translation of *Éléments de géométrie* by Alexis Clairaut.[5]

A later legislative measure introduced in 1858 by Minister Giovanni Lanza definitively formalized the institution in the Kingdom of Sardinia of state schools for training primary teachers (six for men and six for women), now called *Scuole normali*. This model for normal school courses would later be reconfirmed by the Casati Law in 1859. These normal schools also represented the only opportunity

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[4] *Regie Lettere Patenti, 1° agosto 1845*. In *Collezione celerifera delle leggi pubblicate nell'anno 1845 ed altre anteriori* (Torino: Fratelli Favale), pp. 485–508.

[5] See "Decreto per le Scuole di metodo,"*Giornale della Società d'istruzione e d'educazione, 3*, 1851, 199–202 and "Istruzione provvisoria ai signori professori ed assistenti nelle scuole provinciali superiori di metodo," Ibid., 360–368.

open to women for obtaining an education that was formally recognized until almost the end of the nineteenth century.

2.2.2 The creation of the figure of regio professore italiano

The first law to give an overall structure to Italian schools – from primary schools through university – was the Casati Law, from the name of the Minister of Public Education Gabrio Casati who drafted it. Promulgated by King Vittorio Emanuele II on November 13, 1859 (RARS 1859, pp. 1903–1998) to reorganize public instruction in Piedmont and Lombardy, it was gradually extended to the whole country after the Italian Unification in 1861. It constituted the foundation of all Italian school legislation until 1923, when the reform proposed and carried out by the neo-idealist philosopher Giovanni Gentile modified its institutional organization, though leaving some of its fundamental characteristics unaltered.

The Casati Law made the *ginnasio–liceo* the mainstay of Italian schools, provided legal status for public school teachers by defining the requisites for holding the position, and the *concorso* (competitive examination) became the instrument of recruitment of teachers of all subjects.[6] A hierarchy of career stages in three categories – *titolari*, *reggenti*, *incaricati*– was established. The *titolari* were appointed by the King following a competitive examination for which only those who had earned a degree were eligible (Arts. 205 and 206), although the minister could excuse those known for scientific and didactic competence from the competition (Arts. 210 and 263). The category immediately below *titolari* was that of *reggenti*, who were selected from among those who held a legal title for teaching, but had not yet passed a competition. These were nominated for a three-year period at most, after which they could either be promoted to the rank of *titolari*, or have their contract as *reggenti* extended. At the lowest level of the hierarchy were the *incaricati*, appointees who were named from year to year without any qualification and who had no possibilities for advancement. The lack

[6]Actually, the ambiguity of the section of the Casati Law concerning teachers, the existence of secondary schools managed by townships and provinces, in addition to those managed by the State, and the lack of graduates meant that the law was often unfulfilled. The competitive examination, overseen by the Ministry of Public Education, became "national" after Unification, but it was only in 1906 that legislation was approved making it official that only those who had passed a competitive examination (*concorso*) could teach (see §2.3.1). Regulations governing competitive examinations were approved in 1908.

of teachers with a degree often forced schools to fall back on *incaricati*, and this had inevitable repercussions for the quality of teaching.

In particular, mathematics teachers could belong to both the categories of *titolari* and *reggenti*, but Art. 204 allowed *incaricati* to teach arithmetic in the *ginnasio*. The majority of these were priests, physicians and land surveyors who had already taught before. Moreover, starting from1863 special examinations – that were actually amnesties – were introduced to qualify for teaching the various subjects those who had accumulated a certain number of years of teaching experience.

The Casati Law summarily dealt with the normal schools for training primary school teachers, but did not provide for special institutions for training secondary school teachers.

The actual situation of secondary schools emerges clearly from the inquiry undertaken by the High Council for Public Education in 1864. In his well-documented report Giovanni Bertini, professor of philosophy at the University of Turin, underlined the inadequate recruitment of teachers, writing:

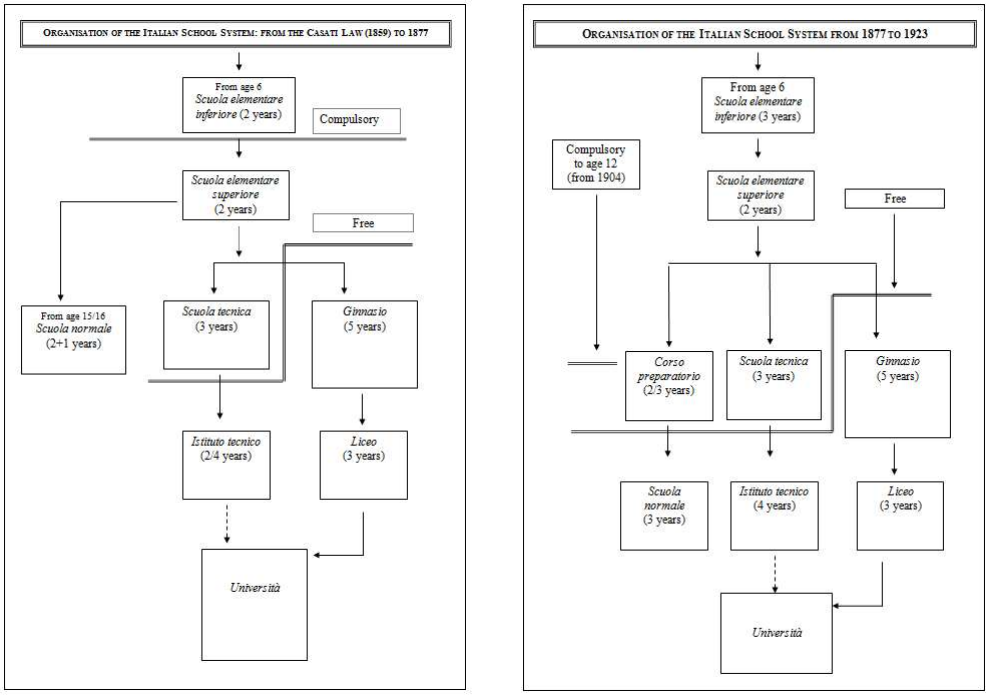
> We have in our schools a small number of teachers who are good, a huge number who are mediocre, and a large number who are incompetent. (Bertini 1865, p. 89)

The reason for this lack of teachers lays implicitly in society's selective mechanism, which was so rigid that those who made it to university rarely chose to study in faculties that led to low-earning careers such as letters, philosophy and mathematics. The dearth of teachers lasted for the whole of the nineteenth century, and only towards its end, when the university came within reach of the lower middle classes, was a certain equilibrium reached between supply and demand for teachers. This situation, in addition to having serious repercussions for the quality of schools, also contributed to the creation of a hierarchy of teachers that was even more complex than that established by the law. Moreover, while *titolari*, with regular degrees, were concentrated in the state *licei* of the large

cities, the teaching staff of the *ginnasi* and above all the technical schools and institutes were often a mixed lot (Ricuperati 1973, pp. 1702–1703).

It should also be remarked that in classical schools the number of hours assigned to scientific subjects was lower by far than that assigned to humanities,[7] and that those teaching them were paid less. The secondary role played by mathematics became even more evident during the final two decades of the nineteenth century because numerous decrees gradually abolished the written test in mathematics from the examinations in ginnasi and licei (Fig. 2).

Table 2 The organization of the Italian school system



2.2.3 The transformation of the SNS of Pisa from a training institute to a research institute

The SNS of Pisa was, at least on paper, the only true school for educating secondary school teachers, but it certainly was not able to meet the needs of secondary school teaching in the new Kingdom of Italy. The importance of creating institutions for training secondary school teachers was strongly felt by the physicist Matteucci, who was professor at the University of Pisa. Once he became Minister of Public Education, he was forced to scale down his aspirations; the

[7]See http://www.subalpinamathesis.unito.it/storiains/uk/tab1.pdf.

only concrete step he was able to take was the decree of August 17, 1862, that modified the Regulations for the SNS of Pisa.[8] The principal innovations consisted in the fact that, in contrast to the earlier *Scuola Normale* of the Grand Duchy of Tuscany, the SNS of the Kingdom of Italy was completely secular, and students from all over Italy could participate in the competitive examination for admission. The course of study was to last three years and students could enter after having attended the first year of university. In addition to university courses, the students at the SNS – both in the section for letters and philosophy, and in that of the physical sciences and mathematics – had to follow particular programs of study and activities within the school, which consisted of:

> 1) special courses established by the Board of Directors, and in practice exercises concerning these courses and university courses as well;
> 2) dissertations on topics assigned in turn by the various instructors and in summaries of the lessons written by the students in their turn;
> 3) lectures regarding the dissertations in which all students are to participate;
> 4) lessons given successively by the various students on topics established by the Board of Directors, sometimes given within the school, sometimes in the *ginnasio* and the *liceo.* (*Notizie storiche* 1871, p. XL)

It is interesting to note that students in the SNS's section of physical sciences and mathematics were required to attend courses in Italian literature, history, geography and two foreign languages, French and either English or German. The examination to obtain the qualification for teaching comprised a dissertation on a topic selected from among the subjects studied during the course of study at the SNS, a public lecture on a mathematical subject from the programs for *licei*, and a demonstration of the candidate's ability to use the instruments of physics and geodesy (*Notizie storiche* 1871, pp. XLVI–XLVIII).

Directing the school for the first three years was the historian and statesman Pasquale Villari, who attempted to combine scientific research with professional training by basing the school on principles of open debate, personal research,

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[8] The text of the Regulations is given in an appendix to *Notizie storiche* (1871).

discussion and exchange of ideas. The educational plan adopted by the SNS was in many aspects similar to that of the seminars that were become widespread in German universities. Villari had studied similar initiatives in other European countries during his travels in France and Great Britain in 1862 and in Germany in 1864 (Moretti 2011, pp. 9–31).

In 1865 Villari was succeeded by Enrico Betti (1823–1892), one of the greatest mathematicians of the Italian Risorgimento (Giacardi, to appear b). Except for two years when he was the Secretary General in the Ministry of Public Education (1874–76), Betti remained director until his death, gradually transforming the SNS from an institute for teacher training into an institute for training and advanced research. Betti strengthened the scientific section and founded the *Annali della Scuola Normale Superiore* for publishing the best theses for qualification. Under Betti's leadership the Pisa School of mathematics quickly acquired international renown, and the SNS was its breeding ground, producing high-level mathematicians such as Ulisse Dini, Salvatore Pincherle, Luigi Bianchi, Vito Volterra, and Federigo Enriques.

2.2.4 Other initiatives for teacher training

Though not many, there were other initiatives aimed at teacher training. The *Istituto tecnico Superiore* of Florence (1867) and the *Scuola di applicazione* for engineers in Rome (1873) attempted to create sections dedicated to teacher training; in 1869 a *Scuola Normale Superiore* was created in Naples with regulations similar to those of Pisa for training teachers at the *ginnasi–licei* (secondary classical school) levels.

In Milan (1863) *Normal* programs were established at the *Istituto Tecnico Superiore* for training teachers for the scientific-technical disciplines, and at the *Accademia scientifico letteraria* for training teachers in the classic-humanistic disciplines. Francesco Brioschi (1824–1897), the driving force behind the creation of the *Istituto Tecnico Superiore*, which would later become the *Politecnico*, affirmed the importance of these courses in his speech at the inauguration ceremony, underlining the fact that they should comprise both the method of study as well as that of research, and should not be limited solely to pedagogical

aspects; moreover, he deplored the separation of the sciences and the humanities, which was always "deleterious to progress" (Brioschi 1863; Lacaita 1984; Decleva 2001). Analogous courses were established at the *Regio Museo industriale* in Turin (1866). However, these schools were not intended to train mathematics teachers (Pittarelli 1902; Ferraresi 2001).

2.3 The Teacher Training Schools: institutional measures, debates and courses

2.3.1 Legislative measures[9]

The *Scuole di Magistero* (university level institutes for teacher training) were established by Minister of Public Education Ruggero Bonghi on October 11, 1875 (GU 1875, 255, pp. 6833–6836) to respond to the need to train future secondary school teachers of the various disciplines. They survived with successive modifications until 1920, when they were abolished by the Minister Benedetto Croce. In 1875–76, out of twenty-one universities, only eight had established *Scuole di Magistero*, while there were only three courses in mathematics, in Pavia, Pisa and Rome. The director of the school in Pavia was Giovanni Cantoni, and the course in mathematics was taught by Felice Casorati; in Pisa the director was Enrico Betti, and the course was taught by Cesare Finzi; in Rome the director was Eugenio Beltrami, and the course was taught by Giuseppe Battaglini (BUMPI 1876, II, pp. 372–373). The history of the *Scuole di Magistero* was especially troubled, as shown by the great number of decrees that concerned them.
The initial purpose of the *Scuole di Magistero* was fundamentally ambiguous, emphasizing an introduction both to research and to professional training (GU 1875, 255, p. 6835).

In order to clarify the aims and to improve their effectiveness, a commission was created in 1885 by the High Council for Public Education, which included Luigi Cremona, Beltrami and Sebastiano Richiardi. The Commission's report placed particular emphasis on the "practical preparation for secondary teaching", up to that time neglected in favour of a specifically scientific preparation, and insisted on the need for practical training dedicated to the study of the foundations of

[9] All the legislative measures cited here are available in the section *Teacher training* of the site http://www.subalpinamathesis.unito.it/storiains/uk/documents.php.

mathematics and critical analysis of the methods.[10] These suggestions were reflected in article 2 of the Royal Decree of December 30, 1888 (P. Boselli), which states that "The *Scuola di Magistero* is aimed at the practical training for secondary teaching" and underlines the importance of this kind of training, which "consists in the examination of the postulates of science, in written works, and in lessons by students on subjects chosen by them at the suggestion of their professor and with his approval. The discussion of the didactic rules to be applied to the aforementioned subject in secondary teaching will be included" (GU 1889, p. 219). Further, in article 4, a certain emphasis was given to mathematics by assigning to it a four-years course, while only two years were assigned to other scientific disciplines.

The nature of the lessons was further defined by Pasquale Villari in the Royal Decree of November 29, 1891, which underlined from the beginning that the primary aim of the courses (of a minimum of 2 years) was to "render the students expert in the art of teaching the different disciplines" (Art. 2) in the various kinds of secondary schools. In particular, article 5 recommended the institution of lectures of a didactic nature to be entrusted only to those who had acquired long years of practical experience in secondary teaching. Article 6 specified that the lecturer should: 1) set forth the method to be used in secondary schools for teaching the subject assigned to him, assessing its extents and limits; 2) require the students to perform appropriate practical exercises to accustom them to applying the method being taught – among these practical exercises there were also actual lessons given in the *Scuole di Magistero*, and, when possible, in a secondary school as well; 3) present and examine the best textbooks for secondary schools (GU 1892, p. 80). This decree provided detailed instructions regarding the methodology of the courses, but it reduced the number of years from four to two, and assigned only one lecture per week given by a single professor.

Successive decrees did not introduce any modifications of note, except for the Orlando Decree of 1903 (GU 1904, p. 24), which made practice teaching in secondary schools optional, thus further deteriorating the *Scuole di Magistero*.

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[10] See *Sull'istruzione secondaria classica. Notizie e documenti presentati al Parlamento Nazionale dal Ministro della Pubblica Istruzione Paolo Boselli* (Roma: Sinimberghi, 1889), pp. 266–269, 273–274.

In many cases the *Scuole di Magistero* were completely inadequate for reliably addressing the problem of teacher training. There were many reasons for this: above all, the professors who taught there were the same ones who taught institutional courses, and because these had, with rare exceptions, no experience in secondary teaching, they were unprepared to address questions about pedagogy and method. Furthermore, supporting structures (libraries, laboratories, etc.) and teaching materials were practically non-existent, the number of assigned course hours was inadequate, and there was scant funding. All of these shortcomings were bemoaned by educators and mathematicians alike.

In 1885, in his report to the Senate on the reorganization of university teaching, Luigi Cremona, at that time a professor at the University of Rome, criticized the lack of pedagogical knowledge on the part of the professors of the *Scuole di Magistero*:

> The great interest that the State has in the formation of qualified teachers demands that these be trained in only a few centres […] under the guidance of men who are not only scientists but also masters of the art of teaching. […] It is precisely in this that the regulations of 1875 were mistaken, in taking too much for granted the pedagogical and didactic preparedness of the professors. (Cremona 1885, p. 85)

In many cases the difficulty of interaction between the university world and secondary teachers was a further reason why the *Scuole di Magistero* were generally inadequate.

It was only in 1906 that legislation was approved regarding the legal status of teachers, making it official that only those who had passed a competitive examination (*concorso*) could teach in the various kinds of secondary schools, and that a degree was required for admission to the competition (GU 1906, 106, p. 2085); it was not until 1914 that mathematics teachers were placed on an equal footing with teachers of Italian, rectifying the inequality that had existed in the system since the Casati Law (RU 1914, III, Art. 1, p. 2391 and Tab. A, p. 2418; Santoni Rugiu 1981, p. 233).

In spite of the problems we have outlined and the inadequacy of the teacher training policy in the period under consideration, it is possible to identify a group of highly qualified and committed mathematics teachers who made a significant contribution to debates on methodology, to the publication of textbooks and of specialized journals for teaching, and to the improvement of legislation regarding education (see Section 3).

In order to give a numeric idea of the context in which secondary teachers worked, it is worthwhile to note that out of the 3,020,200 young people between the ages of 15 and 19 in 1901,[11] about 93,000 attended the various kinds of state secondary schools (lower and upper level), and about 50,000 attended the non-state or private secondary schools.[12] In 1909 mathematics teachers of the state secondary schools in Italy numbered 826, of which 312 taught at *ginnasio–liceo* (BUMPI 1909, pp. 894–932).

2.3.2 Debates at the associations of teachers and the mathematics courses in the Scuole di Magistero

From the very beginning, the problem of the professional training of teachers was one of the most hotly debated topics in the *Associazione Mathesis*, an association of teachers of mathematics founded in Turin in 1895 by three secondary teachers, Rodolfo Bettazzi, Aurelio Lugli and Francesco Giudice, with the aim of "improving the school and the training of teachers, from the points of view of science and didactics."[13] At its first congress, held in Turin in 1898, *Mathesis* sponsored an enquiry among its members regarding the theme "Modifications to be introduced in the regulations of university mathematical studies, intended to produce good secondary teachers." The theme was taken up again in subsequent meetings and congresses.[14]

Within the multiplicity of proposals that were formulated, it is possible to identify two lines of thought. Some, such as Salvatore Pincherle, proposed the separation

[11] The total population in Italy in 1901 was 32,475,200 people. See Istituto Centrale di Statistica, *Sommario di statistiche storiche dell'Italia 1861–1975*, Roma 1976, p. 12.
[12] See Commissione Alleata in Italia 1947, Tab. V, VI, XIII, XIV.
[13] See Statuto dell'Associazione 1896, p. 161.
[14] See Nastasi (2002), and Appendix 1.

of formative courses, and more precisely, the institution, after the first two years of university, of a special school leading to a degree in education (*laurea didattica*) to be attended by all those who intended to pursue a career in secondary teaching; this was to be distinct from the degree in pure mathematics, intended instead for those who wished to pursue a career in research (Pincherle 1906, p. 86). Others, such as Alessandro Padoa, supported by Gino Loria and Giuseppe Peano, disapproved of the separation into two formative courses, and instead believed that it was urgent to strengthen the *Scuole di Magistero* (Loria and Padoa 1909). For further details see Appendix 1.

No systematic study of the courses held by mathematicians in the *Scuole di Magistero* has yet been carried out, but archival research has brought to light some significant information about them. In particular the courses given at the University of Turin by Corrado Segre (1863–1924),[15] the father of the Italian school of algebraic geometry, in Bologna by Cesare Arzelà (1847–1912) and Salvatore Pincherle (1853–1936),[16] accomplished mathematicians in the field of analysis, and in Rome by Giulio Pittarelli (1852–1934),[17] a secondary school teacher and later professor at the university, were all characterized by: the importance attached to elementary mathematics from an advanced standpoint; growing interest in questions regarding the principles and foundations of mathematics; attention to questions of methodology and didactics; and the influence of Klein. From archival documents, in spite of gaps, it is possible to learn the number of men and women in attendance, and to note the progressive increase in the number of women. Documentation regarding these courses is found in Appendix 2.

2.4 From the suppression of the *Scuole di Magistero* to the Gentile reform

On October 8, 1920, the *Scuole di Magistero* were abolished by Croce (BUMPI 1920, p. 2064), an important representative of Italian Neo-idealism, who maintained that educational training and scientific training were one and the same

[15] See ASUT, *Conferenze della Scuola di Magistero di Scienze*, VII 84, and Giacardi (2010).
[16] See ASUB, *Scuole di Magistero* (pos. 53/b), busta 2 (1893–1908) and ASUB, *Scuole di Magistero* (pos. 53/b), busta 3 (1880–1921).
[17] ASUR, *Facoltà di Scienze. Libretti delle lezioni: Pittarelli Giulio.*

thing. Some of the most vigorous opposition came from *Mathesis* and the two members of the Italian school of geometry, Loria and Gino Fano. Loria expressed indignation for this "sudden and violent measure", saying that the *Scuole di Magistero* represented "a bridge, the only one that exists between upper-level and middle-level teaching." He criticized the identification of scientific training with educational training, the lack of interest in questions of methodology, and the fact that "future teachers were not put in front of school students in the way the future health worker is put in contact with human suffering" (Loria 1921, p. 163).

During the 1921 *Mathesis* congress in Naples, Fano asked for "the reinstatement of the *Scuole di Magistero* for mathematics, in a broader and more comprehensive form than the previous one." Convinced that "*knowing more* than what you teach is worthless, if this *more* does not make you know better what should be taught", he energetically proposed the establishment of courses of elementary mathematics from an advanced standpoint, with an emphasis on the historical, critical, methodological and didactical aspects, citing the lessons of Segre and Enriques as examples. He also invited the faculties to accept degree dissertations in those sectors of mathematics more strictly connected to elementary mathematics, and urged his colleagues to establish, without awaiting ministerial decrees, practice teaching programs in secondary schools for the future teachers (Fano, 1922, pp. 103, 109).

2.4.1 The institution of the combined degrees and the courses in complementary mathematics

The proposals were accepted at least in part by the Minister of Public Education Orso Mario Corbino, who on November 24, 1921, established "combined" degrees (*lauree miste*) in physical and mathematical sciences (BUMPI 1922, p. 22) aimed at qualifying young people to teach scientific subjects in secondary schools, and on February 19, 1922, instituted a course in complementary mathematics (*matematiche complementari*), accompanied by didactic and methodological exercises (BUMPI 1922, p. 349).

The decree that instituted the combined degree in physical sciences and mathematics also specified that the degree dissertation be substituted by a

practical examination in physics, a written and an oral examination in general knowledge, and a public lecture on a topic chosen from among those in the secondary school programs. This measure was not favourably received by all mathematicians. In Rome, for example, Volterra, Alfonso Di Legge, Pittarelli and Guido Castelnuovo in a report presented to the faculty in March 1922[18] expressed their concern that the new way opened by the combined degree might lower the level of the education of future secondary school teachers of mathematics and physics, and noted:

> This course [complementary mathematics], useful, as said earlier, for coordinating notions of higher mathematics acquired in other courses, is certainly no substitute for these. And even if it were possible to reduce it to a mosaic of fragmentary information about analysis, geometry, mechanics, it could still never replace those systematic courses, which must show young people how science is constructed and has developed: it is aimed at those who, even while unable to contribute to that development, must know something about the most advanced branches of the science whose elements they are to teach. (Gario 2004, p. 118)

In contrast, Tullio Levi-Civita, Enriques and Francesco Severi deemed that such a course was sufficient to "fill the gaps in education left unfilled by the courses of the first biennium and provide sufficient preparation in mathematics for future teachers" (Gario 2004, p. 119). The following year the Gentile Reform would put a stop to any further initiatives of an institutional type regarding teacher training.

It is possible to form an idea of the nature of the courses in complementary mathematics from a preliminary investigation of the programs carried out in some Italian universities[19] and from the various initiatives promoted by some mathematicians (Enriques and Peano, among others) for teacher training. What emerges most clearly is the attention paid to elementary mathematics from an

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[18] This document is transcribed in Gario (2004).

[19] See for example: the register of lessons and exercises (1922–1923) of Enriques in Giacardi (to appear a: *Appendix* 1.2, 3); ASUR, *Facoltà di scienze, Libretti delle lezioni, 1923–1926: Castelnuovo Guido, Perna Alfredo*; ASUT, Ufficio posta e protocolli, Università di Torino, *Corrado Segre*; Luciano and Roero (2008).

advanced standpoint, to the history of mathematics, and sometimes to problems of didactics and methodology as well. For further details see Appendix 3.

In 1923, Gentile, then Minister of Public Education, taking advantage of the full powers given to him by the first Fascist government, carried out a complete and radical reform of the school system. Humanistic disciplines were to form the main cultural axis of national life and of education in particular, and the sciences were strongly penalized. Protests were of no avail: not those of the mathematicians; not those of the scientific associations (Giacardi, 2006, pp. 54–63). Moreover, Gentile, identifying "knowing" with "knowing how to teach", believed that teacher training merely consisted in "genuine, profound and authentic scientific preparation" (Gentile, 1907, pp. 178–179) and did nothing for the professional development of teachers. The course in complementary mathematics thus constituted the only aid given to future mathematics teachers.

The report prepared by the ministerial supervisor Alfredo Perna on teacher training in Italy for the ICMI section of the International Congress of Mathematicians in Zurich (1932) sheds light on the Italian situation regarding teacher training under the Fascist dictatorship (Perna 1933). He lists the institutional shortcomings in this area: no institutes for professional teacher training; no courses on methodology and pedagogy at the universities; no scholarships designated for teacher training; life-long learning was not compulsory, training courses for in-service teachers being left up to individuals. Only the initiatives on the part of some mathematicians compensated to a certain extent for the institutional shortcomings.

3 Mathematics teachers in the unified Italy[20]

Following Schubring (1984) we maintain that, while programs and Ministry circulars provide the necessary elements for analyzing the dominant conceptions about mathematics teaching, other elements, such as manuals and teachers themselves, better illustrate teaching as it is actually carried out in the classroom. In this section we look at these elements to outline aspects of the profession of

[20] By Fulvia Furinghetti.

mathematics teaching in the decades from Italian Unification up to the Gentile reform.

Mathematics teachers, together with primary teachers and teachers of other disciplines, contributed fundamentally to the process of unification in Italy, since the system of recruitment based on national competitions fostered a remarkable mobility of teachers, who often obtained their first post in places far from their home. This mobility, unusual in Italian society (then as now), promoted contacts among people with different customs, dialects, and backgrounds in the various regions.

There was a fertile exchange between universities and secondary schools: university professors often began their careers as secondary school teachers and some secondary teachers delivered regular courses at the university, though not having a permanent position.

The great ferment of initiatives in the world of schools was not only fostered by the general atmosphere of enthusiasm during the construction of the nation, but also by the state's concrete acknowledgements of distinguished teachers: upgrades in salary, advancements in career, distinctions, awards for the best treatises (on methodologies of mathematics teaching, or on the presentation of mathematical topics).

The landmarks in the development of the community of mathematics teachers towards a professionalization are:

- The production of new textbooks. Some of these were so good that they were translated into foreign languages. Smith (1900, p. 304) claims that "Italy has produced some excellent works on elementary geometry; indeed, in some features it has been the leader." Also Klein (1925–1933, II) remarked on this phenomenon in the chapter *Der Unterricht in Italien*;
- The regular publication of journals for mathematics schoolteachers or for secondary students;

- The creation of teacher associations: in 1895 *Mathesis*, and in 1901 FNISM (*Federazione Nazionale Insegnanti Scuola Media*)
- , a professional association of teachers founded with aims of protecting economic and legal rights of teachers of different disciplines.

In comparing the dates of analogous events (local production of textbooks, foundation of journals and associations) in other countries we realize that at the end of nineteenth century Italy was well placed relative to the most important countries. From the ranks of ordinary teachers whose stories are obscured by time, there emerge some eminent figures who left behind evidence and documents that shed light on their contributions to the development of the community of mathematics teachers. They published articles in didactic journals, and textbooks (original or translated), promoted communication and cooperation through associations and conferences, and were involved in discussions of policy concerning the status of mathematics and of the teaching profession in the system of national instruction. Some teachers also made important scientific contributions in mathematical research and so deserve a place in the history of mathematics.

There are various sources for identifying these outstanding teachers. A first, partial, source is the list compiled by Tricomi (1962), which contains short biographical notes on 371 Italian mathematicians (of various degrees of importance for their mathematical research) who died in the period January 1, 1861, to December 31, 1960. About 100 of these had been secondary teachers for all or part of their career (see Appendix 4).

A further source is the list of the members of *Mathesis*, which testifies to the existence of a number of mathematics teachers who showed a major involvement in their profession through their support of the association. In 1914, of the 467 members, not counting university teachers and supporters of various kinds, about 350 were secondary teachers. The involvement ranged from a simple acknowledgment of belonging to a community through payment of dues, to taking an active part in the debates on curricula and on teaching methods, to regular participation in regional and national meetings. The reports of the regional meetings of *Mathesis* Association supply further information on the nature of this

involvement (Furinghetti 2002). The universe of teachers who contributed to the development of mathematical education in Italy also included women quite early. In spite of the limitations of laws and prejudices in society and among colleagues, slowly women became the backbone at first of Italian primary and later of secondary education (see Appendix 5).

Professional journals are the best sources for identifying teachers who contributed to the evolution of mathematics teaching. The articles, notes and letters to the editors reveal the most active people and what they gave to the mathematics education community.

Taking as a guiding thread the three elements that have characterized the innovations of school mathematics – that is, national production of textbooks, founding of journals, and the founding of and participation in associations – in what follows we portray some of the different forms of commitment to the profession of mathematics teaching during the period in consideration.

3.1 Teachers and textbooks

An area in which Italian mathematics teachers showed a remarkable competence was the production of textbooks. Before Unification, in the galaxy of states in the Italian territory a variety of different kinds of textbooks were used. Of primary importance were the French textbooks by Clairaut, Lacroix and Legendre, which enjoyed many translations and inspired several treatises, but there were also books by local authors which were not always of the highest level (Pepe 2006). An updating of the textbooks started with the translation of manuals that appeared around the middle of the century. In 1869 Achille Sannia and Enrico D'Ovidio published *Elementi di Geometria*, a book that was aimed at adapting the 1867/1868 edition of Euclid's *Elements* by the mathematicians Betti and Brioschi to suit the needs of schools. Two later books by Riccardo De Paolis (1884) and by Giulio Lazzeri and Anselmo Bassani (1891) were based on the fusion of plane and solid geometry. Along with the group of works which closely followed the Euclidean pattern, there began to appear a new generation of textbooks that were influenced by the flourishing of research in the foundations of mathematics.

3.1.1 Aureliano Faifofer

Among the most prominent authors of textbooks is Aureliano Faifofer (Borgo Valsugana, August 4, 1843 – Venice, March 1, 1909).[21] He is representative of those teachers who after a short initial activity in university espoused the career of secondary mathematics teacher completely and with conviction. He attended secondary school and university in Padua, graduating in mathematics in 1863 under the advisement of Giusto Bellavitis. Initially he was assistant professor in this university, but in 1868 he obtained the position of mathematics teacher at the Liceo Foscarini in Venice, where he taught until his death. Among his students there were two important Italian mathematicians: Guido Castelnuovo and Guido Fubini.

Faifofer's biography illustrates clearly his vocation for teaching. He began his career just after Unification, living in that phase, full of enthusiasm and lively debates, during which the educational system of the new nation was being constructed. He interpreted his mission as providing the important means suitable for establishing new methods for mathematics teaching, that is, reliable textbooks. His *Elementi di geometria* (Venezia, 1878) had a number of editions and was a point of reference for generations of mathematics teachers due to its pedagogical clarity and its care for mathematical concepts; the treatment of equivalence theory is particularly noteworthy and influenced the textbooks that followed. It was translated into foreign languages. In Faifofer (1907) the author reports the enthusiastic comments about his manual by Beltrami, Paul Mansion, and W. W. Beman. Faifofer also wrote treatises on algebra, tables of logarithms and trigonometric functions. Although less well known, his textbooks on arithmetic are also interesting for their rational approach to arithmetic and real numbers (Menghini, to appear). After his death Faifofer's textbooks were slowly replaced by those by Enriques and Ugo Amaldi. In the introduction of the first edition of 1903 these authors acknowledge Faifofer as an "outstanding teacher" (*didatta insigne*) and their debt to him.

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[21] See Ciamberlini (1909); Il Consiglio Direttivo (1909).

3.2 Mathematics teachers and journals

The first mathematical journals, published from 1850 on, were not specifically aimed at school teaching, and soon the need for having journals for this purpose was felt by Italian mathematics teachers (see Furinghetti and Somaglia 1992). The founding of such journals contributed to the communication within the country, and became a means of spontaneous in-service training. They also created the milieu suitable to for the establishment of *Mathesis*.

3.2.1 An early didactic journal

The first Italian journal specifically devoted to mathematics teaching was *Rivista di Matematica Elementare*, founded in Alba by Giovanni Massa[22] (Alba, May 9, 1850 – Milan, April 8, 1918). Massa was a teacher of accounting and bookkeeping in technical institutes for accountants. His professional interests were mainly in the field of accounting; he contributed to the development of this discipline in Italy at the end of nineteenth century and carried out important editorial initiatives in this field. The main themes of the journal were arithmetic, elementary number theory, combinatorial calculus, algebra, and geometry. Only a few articles dealt with analysis, history of mathematics, and foundations of mathematics. The authors and the readership were mainly secondary teachers, and a few university professors. Aspects of Massa's life epitomize the spirit that animated his enterprises in education. He showed a great interest in social issues: he was involved in local association of workers, and supported the universal suffrage, education that was public, secular and compulsory, freedom of the press and freedom of assembly. The mission of his journal was to be the catalyst for the ferments animating mathematics teachers during those years and for spreading new ways of teaching throughout Italy. As a matter of fact, among the journal's collaborators were some teachers who played a very active role in the restyling of mathematics teaching, such as: Alberto Cavezzali (1848–1922), editor of *Il Piccolo Pitagora* in Novara (Piedmont), issued in 1883–84; Pietro Caminati (1837 – after 1928), editor of *Il Tartaglia* in Foggia (South Italy), issued in 1898–99; and Davide Besso (1845–1906), the founder of *Periodico di Matematica*, in Rome.

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[22] See D'Alterio (2008).

Publication of Massa's journal stopped in 1885, but the seed had been sown: a year later the *Periodico di Matematica* was founded. In 1895 the journals *Il Pitagora* (for students) and in 1902 *Il Bollettino di Matematica* (for mathematics teachers) appeared. Their editors were secondary teachers, whose work mirrors the missions and values of their times.

3.2.2 Gaetano Fazzari

Gaetano Fazzari[23] (Tropea, October 7, 1856[24] – Messina, July 13, 1935), after having earned his *laurea* in mathematics (Naples, 1884), taught in various places in Southern Italy. In the 1898–99 school year he obtained a permanent position as a teacher in the Liceo Umberto I of Palermo, where he stayed until the end of his career (1926). For some years he also acted as vice-principal of this school. He was a much esteemed teacher and received official acknowledgments from the Ministry of Education: two promotions for distinguished merit, inscription in the honor roll of secondary teachers, and appointment as an Inspector for mathematics teaching.

Fazzari's main interests in mathematics were geometry (in particular conics, which were the subject of his dissertation and on which he published papers in *Giornale di Matematiche*), elementary arithmetic and history of mathematics. He authored articles on the teaching of mathematics, among them a report for the *Commission Internationale pour l'Enseignement Mathématique*, regarding mathematics teaching in classical schools (Fazzari 1911).

History was his main cultural interest and played an important role in his conception of mathematics teaching. It also animated the journal he founded, *Il Pitagora*. This journal, addressed to secondary school students, was popular from its very first issues, and by raising questions and asking for opinions, it became a kind of forum on the problems and expectations in schools during the two decades at the turn of the century. With few exceptions (one being the historian Gino

[23] See Borrello (1935); Furinghetti (2000); Natucci (1939).
[24] Some official biographies report 6 as the date of the birth, but 7 is the date carved on the tomb.

Loria), the contributors were secondary teachers. The first issue appeared in 1895, the last in December 1918.

3.3 Mathematics teachers and the association *Mathesis*

When *Mathesis* was first founded, it was only open to secondary teachers (Statuto dell'Associazione 1896). In 1908 university professors were also allowed to join the association, and soon some of them served as president. The association organized national congresses, where the reforms of the beginning of the twentieth century were discussed. Regional branches were established for smaller meetings. A bulletin was issued, sometimes as a separate brochure, sometimes inserted in *Periodico di Matematica*. The association underwent many changes in spirit and rules, but still exists today with the name *Mathesis, Società italiana di scienze fisiche e matematiche*, adopted in 1922.

3.3.1 Francesco Giudice

The mission and vision of *Mathesi*s in its early days are epitomized by the lives of its founders, who combined competence in mathematics with a view of their profession based on communication. We present one of them, Francesco Giudice (Codevilla, March 1, 1855 – Voghera, August 11, 1936),[25] who also represents those teachers who taught regularly in university. He earned a *laurea* in engineering at the University of Turin in 1877 and another in mathematics at the University of Pavia in 1881. He taught in various secondary schools, the most important being the royal technical institutes in Genoa and Pavia. He earned his *libera docenza*, a special qualification delivered on the basis of an oral examination and scientific publications that allowed the holder to teach at university. He taught at the University of Pavia starting in the 1911–12 academic year until his retirement in 1925. When the course of complementary mathematics was inaugurated at the University of Pavia, he was appointed as a lecturer. His interest in teacher training is shown by the publication of a university textbook for prospective teachers.

In the early years of his career Giudice participated in the activities of Peano's group. He published an Italian translation of a book by Klein that was important

[25] *Annuari dell'Università di Pavia* 1911–12 to 1924–25, and Conti (1936).

for teacher training, *Vorträge über ausgewählte Fragen der Elementargeometrie*, see (Klein 1996). He authored papers in algebra and analytic geometry and textbooks for secondary schools (in algebra, geometry, trigonometry and descriptive geometry).

His mission to connect people is demonstrated not only by the founding of *Mathesis*, but also by his constant collaboration with the journals devoted to mathematics teaching that we mentioned earlier.

4 Conclusions

The process of Italian unification was undoubtedly one of the most important factors for the growing awareness on the part of the community of Risorgimento mathematicians of the importance of training teachers in order to create secondary schools that were efficient and capable of preparing the future ruling class of the young nation. This led to the creation of the *Scuole di Magistero*. Their small number, the reduced number of lesson hours, the lack of funding, and the continual conflicts between those who saw teacher training as purely scientific and those who maintained the importance of professional training made them generally inadequate for the purpose. These factors led to their eventual suppression, with the consequence that the problem of teacher training remained unresolved until the end of the twentieth century.

In spite of the inadequacy of a policy for teacher training, the flourishing of Italian mathematical research and the atmosphere of activism and enthusiasm created by the birth of the new nation influenced the school world and stimulated interesting initiatives. The publication of good textbooks and the founding of journals devoted to mathematics teaching, the creation of an association for Italian mathematics teachers, and the national and local meetings fostered debates on methodology, school legislation and the teaching profession. Along with mathematicians, a committed group of highly qualified secondary school teachers participated in these initiatives, as illustrated by the three outstanding figures we have presented.

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Abbreviations

ASUB: Archivio Storico dell'Università di Bologna
ASUR: Archivio Storico dell'Università di Roma
ASUT: Archivio Storico dell'Università di Torino
BUMPI: *Bollettino Ufficiale del Ministero della Pubblica Istruzione*
GU: *Gazzetta Ufficiale del Regno d'Italia*
RARS: *Raccolta degli Atti del Governo di Sua Maestà il Re di Sardegna*
RU: *Raccolta Ufficiale delle leggi e dei decreti del Regno d'Italia*

References

Bertini, G. (1865). Relazione e proposte sull'Istruzione Secondaria. In *Per la riforma delle scuole medie. Scritti vari* (pp. 81–114). Torino: Scioldo, 1889.

Borrello, L. (1935). Gaetano Fazzari. *Annuario R. Liceo Umberto I* (Palermo).

Brioschi, F. (1863). L'Accademia scientifico-letteraria e l'Istituto tecnico-superiore di Milano. *Rivista Italiana di Scienze, Lettere ed Arti colle Effemeridi della Pubblica Istruzione, IV*, 785–789.

Bucci, S. (1976). *La scuola italiana in età napoleonica*. Roma: Bulzoni.

Capello, L. (1917–1918). La prime scuole di metodo del Piemonte. *Atti della R. Accademia delle Scienze di Torino, 53*, 390–408.

Cardinali, V. G., Antonetto, L., & Primosich, F. (1983). *Vincenzo Troya. Vita e opere di un educatore piemontese.* Magliano Alfieri (Cuneo): Pro Loco.

Ciamberlini, C. (1909). Necrologio Aureliano Faifofer. *Il Bollettino di Matematica, 8*, 128–132.

Commissione Alleata in Italia (1947). *La politica e la legislazione scolastica in Italia dal 1922 al 1943 con cenni introduttivi sui periodi precedenti e una parte conclusiva sul periodo postfascista*. Milano: Garzanti.

Conti, A. (1936). Necrologio. Francesco Giudice. *Il Bollettino di Matematica, n. s., 15*, 88.

Cremona, L. (1885). Modificazioni alle leggi vigenti per l'istruzione superiore del Regno. Relazione, 15.3.1885, *Atti parlamentari. Senato del Regno* (N. 100-A) (pp. 1–90). Roma: Forzoni, Tip. del Senato.

D'Alterio, D. (2008) Giovanni Massa. *Dizionario biografico degli italiani, 71*. Rome: Istituto della Enciclopedia italiana.

Decleva, E. (2001). Una facoltà filosofico-letteraria nella città industriale alla ricerca di un'identità (1861–1881). In G. Barbarisi, E. Decleva, & S. Morgana (Eds.), *Milano e l'Accademia scientifico-letteraria* (pp. 3–196). Milano: Cisalpino.

Faifofer, A. (1907*). Elementi di geometria ad uso dei Licei. (12^a edizione ricavata dalla 15^a edizione della stessa opera dedicata ai licei ed agl'Istituti Tecnici)*. Venezia: Sorteni & Vidotti.

Fano, G. (1922). Le scuole di Magistero. *Periodico di matematiche, 4*(2), 102–110.

Fazzari, G. (1911). L'insegnamento delle matematiche nelle Scuole classiche. II. Critiche e proposte. *Bollettino della "Mathesis". Supplemento. Atti della sottocommissione italiana per l'insegnamento matematico, 3*, 35–48.

Ferraresi, A. (2001). Museo industriale e Scuola di applicazione per gli ingegneri: alle origini del Politecnico. In U. Levra (Ed.), *Storia di Torino 7, Da capitale politica a capitale industriale (1864–1915)* (pp. 793–835). Torino: Einaudi.

Furinghetti, F. (2000). The long tradition of history in mathematics teaching: an old Italian case. In V. Katz (Ed.), *Using history to teach mathematics: An international perspective*. MAA Notes n. 51 (pp. 49-58). Washington, DC: MAA.

Furinghetti, F. (2002). Il *Bollettino della Mathesis* dal 1909 al 1920: pulsioni tra temi didattici internazionali e nazionali. *PRISTEM/Storia, 5*, 31–58.

Furinghetti, F., & Somaglia, A. (1992). Giornalismo matematico 'a carattere elementare' nella seconda metà dell'Ottocento. *L'insegnamento della Matematica e delle Scienze Integrate, 15*, 815–852.

Gario, P. (2004). Guido Castelnuovo e il problema della formazione dei docenti di matematica. *Supplemento ai Rendiconti del Circolo Matematico di Palermo, 2, 74*, 103–121.

Gentile, G. (1907). La preparazione degli insegnanti medi. In H. A. Cavallera (Ed.) (1988), *La nuova scuola media* (pp. 161–268). Firenze: Le Lettere.

Giacardi, L. (Ed.) (2006). *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*. Lugano: Lumières Internationales.

Giacardi, L. (2010). The Italian school of algebraic geometry and mathematics teaching in secondary schools. Methodological approaches, institutional and publishing initiatives. *International Journal for the History of Mathematics Education, 5*(1), 1–19.

Giacardi, L. (to appear a). Federigo Enriques (1871–1946) and the training of mathematics teachers in Italy. In S. Coen (Ed.), *Mathematicians in Bologna 1861–1960.* Basel: Birkhäuser.

Giacardi, L. (to appear b). "Pel lustro della Scienza italiana e pel progresso dell'alto insegnamento" L'impegno dei matematici risorgimentali. In A. Ferraresi & E. Signori (Eds.), *Le Università e l'Unità d'Italia (1848–1870).* Bologna: Clueb.

Il Consiglio Direttivo (1909). Aureliano Faifofer. *Bollettino della "Mathesis", 1*(1), 14–15.

Klein, F. (1896). *Conferenze sopra alcune questioni di geometria elementare*. Traduzione di Francesco Giudice. Torino: Rosenberg & Sellier. Originally *Vorträge über ausgewählte Fragen der Elementargeometrie*. Leipzig: B. G. Teubner, 1895.

Klein, F. (1925–1933). *Elementarmathematik vom höheren Standpunkte aus*, I *Arithmetik, Algebra, Analysis*, II *Geometrie*, III *Präzisions und Approximationsmathematik.* Berlin: Springer.

Lacaita, C. G. (1984). *Sviluppo e cultura. Alle origini dell'Italia industriale*. Milano: F. Angeli.

Loria G. (1921). La preparazione degli insegnanti medi di matematica. *Periodico di Matematiche, 4*, 149–164.

Loria, G., & Padoa, A. (1909). Preparazione degli insegnanti di matematica per le scuole medie. In *Atti del II° congresso della "Mathesis" Società Italiana di Matematica*, Allegato A: 1–10. Padova: Premiata Società Coperativa.

Luciano, E., & Roero C. S. (Eds.) (2008). *Giuseppe Peano Matematico e Maestro.* Torino: Dipartimento di Matematica.

Menghini, M. (to appear). Rational approaches to arithmetic and real numbers in the Italian textbooks and programmes for the classical and the scientific curricula. In *Proceedings of ICHME-2*.

Moretti, M. (2011). Toscana, Italia, Europa: la Normale di Pisa e i modelli universitari fra Otto e Novecento. *Annali della Scuola Normale Superiore di Pisa. Classe di Lettere e Filosofia*, *s. 5, 3*(1), 11–33.

Nastasi, P. (2002). La Mathesis e il problema della formazione degli insegnanti. *PRISTEM/Storia*, *5*, 59-119.

Natucci, A. (1939). Necrologio [Fazzari]. *Il Bollettino di Matematica*, *s. 3*, *1*, XII–XIV.

Notizie storiche sulla R. Scuola Normale Superiore di Pisa, *Annali della R. Scuola Normale Superiore di Pisa. Scienze Fisiche e Matematiche*, *1*, 1871, I–XLVIII.

Pagano, E. (2008). I licei italici tra iniziativa statale e realtà urbane. In E. Brambilla, C. Capra & A. Scotti (Eds.), *Istituzioni e cultura in età napoleonica* (pp. 454–474). Milano: F. Angeli.

Pepe, L. (2006). Insegnamenti matematici e libri elementari nella prima metà dell'Ottocento: modelli francesi ed esperienze italiane. In L. Giacardi (Ed.), *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia* (pp. 65–98). Lugano: Lumières Internationales.

Perna, A. (1933). Preparazione dei candidati all'insegnamento della Matematica in Italia. *L'Enseignement Mathématique*, *32*, 245–254.

Pincherle, S. (1906). Sulle riforme scolastiche da compiersi e in particolare su quelle relative all'insegnamento della matematica. *Il Bollettino di Matematica*, *5*(5–6), 83–87.

Pittarelli, G. (1902). Modificazioni da introdursi nell'insegnamento matematico superiore per la preparazione degl'insegnanti secondari. *Atti del Secondo Congresso dei Professori di Matematica delle scuole secondarie* (pp. 137–164). Livorno: Giusti.

Ricuperati, G. (1973). La scuola nell'Italia unita. In *Storia d'Italia*, Vol. 2, I Documenti (pp. 1693–1736). Einaudi: Torino.

Santoni Rugiu, A. (1981). *Il professore nella scuola italiana.* Firenze: La Nuova Italia.

Schubring, G. (1984). Essais sur l'histoire de l'enseignement des mathématiques, particulièrement en France et en Prusse. *Recherches en Didactique des Mathématiques*, *5*, 343–385.

Sideri, C. (1999). *Ferrante Aporti, sacerdote, italiano, educatore*. Milano: F. Angeli.

Smith, D. E. (1900). *The teaching of elementary mathematics*. New York, London: Macmillan.

Statuto dell'Associazione (1896). *Bollettino dell'Associazione Mathesis*, *1*, 1896–97, in *Periodico di Matematica*, 1896, 161–172.

Tomasi, T., & Sistoli Paoli, N. (1990). *La Scuola Normale di Pisa dal 1813 al 1945. Cronache di un'istituzione*. Pisa: ETS.

Tricomi, F. G. (1962). Matematici del primo secolo dello Stato unitario. *Memorie dell'Accademia delle Scienze di Torino, Classe Scienze Fisiche, Matematiche, Naturali, s. 4, 1*, 1–120.
Vidari, G. (1927). Lettere di Piemontesi a Ferrante Aporti nel periodo 1838-1849. *Atti della R. Accademia delle Scienze di Torino, 42*, 650–708.
Weech, W. N. (Ed.) (1945). *History of the World*. London: Odhams.

Appendix 1

Debates at the associations of teachers regarding teacher training[1]

From the very beginning, the problem of the professional training of teachers was one of the most hotly debated topics in the *Associazione Mathesis*, an association of teachers of mathematics founded in Turin in 1895-1896 by Rodolfo Bettazzi, Aurelio Lugli and Francesco Giudice with the aim of "improving the school and the training of teachers, from the points of view of science and didactics."[2] From the association's very first congress, held in Turin in 1898, the Mathesis Association sponsored an enquiry among the members regarding the theme "Modifications to be introduced in the regulations of university mathematical studies, intended to produce good secondary teachers." The theme was taken up again in subsequent meetings and congresses, and different proposals were formulated.[3]

Within the multiplicity of presentations, in substance it is possible to identify two lines of thought. Some, such as Salvatore Pincherle (Bologna, 1903), proposed the separation of formative courses, and more precisely, the institution, after the first two years of university, of a special school leading to a degree in education (*laurea didattica*) to be attended by all those who intended to pursue a career in secondary teaching; this was to be distinct from the degree in pure mathematics, which was instead to be sought by those who intended to pursue a career in research. According to Pincherle, the future teachers should, in a biennial course dedicated to mathematical methodology, inspect and analyze in depth all of the chapters of elementary mathematics (Pincherle, 1906, p. 86). This proposal was supported by Guido Castelnuovo, as well as by Enriques, whose position on institutional ways of providing an adequate scientific and educational training for teachers emerges from the report prepared on the occasion of the fifth congress of *Federazione Nazionale Insegnanti Scuola Media* in 1906. In a rather long introduction, he presented his vision of scientific teaching, and his idea of a

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[1] By Livia Giacardi.
[2] See "Statuto dell'Associazione", *Bollettino dell'Associazione Mathesis*, 1, 1896–97, in *Periodico di Matematica*, 1896, p. 161.
[3] See Nastasi (2002), Gario (2006), and the section *Mathesis Congresses* in http://www.subalpinamathesis.unito.it/storiains/uk/documents.php.

philosophical university based on the German model, then suggested the establishment of a *pedagogical degree* in addition to the *scientific degree.* According to Enriques, it was important to offer future teachers courses on those parts of science that aim at a more profound understanding of the elements, lectures devoted to concrete questions of pedagogy and to the analysis of the textbooks. In addition, they should practice teaching, partly in university classes, and partly in secondary schools (Enriques 1907, p. 78).

Others, such as Alessandro Padoa, supported by Gino Loria and Giuseppe Peano, disapproved of the separation into two formative courses, and instead believed that it was urgent to strengthen the *Scuole di Magistero* (Florence Congress, 1908). In particular, they proposed instituting, in addition to an obligatory period of practice teaching in a secondary school, a two-year course in "mathematical methodology" in place of the didactic lectures in the *Scuole di Magistero*, which would make it possible to address not only topics of arithmetic, algebra and geometry useful for the future teacher, but also include an examination of teaching methods, an analysis of school textbooks, and demonstrate the educational usefulness of the history of mathematics and mathematical games (Loria and Padoa 1909). Loria and Padoa wrote that the history of mathematics should permeate the entire program, aiming above all to reconstruct the various phases of development of each theory, as well as to render the subject less arid and more attractive. In their words:

> The new university course we are suggesting would serve, in our opinion, to fill the deplorable abyss that separates university teaching from secondary teaching today, … which F. Klein has recently referred to as "a system of double forgetting" [n.b. *doppelte Diskontinuität*]: the university student's forgetting what he studied in secondary school, and the secondary school teacher's forgetting all that he studied while he was at university. (Loria and Padoa 1909, pp. 3–4)

References to Klein emerge in all the Mathesis congresses, and are an index of the influence he exerted in the Italian debates, an influence that can be also perceived at the base of the project for the *Enciclopedia delle matematiche elementari* presented by Luigi Berzolari and Roberto Bonola during the congress in Padua in 1909 and completed only in the 1940s. Intended for mathematics teachers as well

as the students of the *Scuole di Magistero*, the encyclopaedia was intended to address elementary mathematics from an advanced standpoint as well as contain suitable remarks regarding the history of mathematics and questions of education (Berzolari and Bonola 1909). With the same aim, in 1900 Enriques had published his famous collective volume, also clearly influenced by Klein, entitled *Questioni riguardanti la geometria elementare*, successively broadened into the *Questioni riguardanti le matematiche elementari* (1912). The topics treated were congruence, equivalence, the parallel theory, problems that could or could not be solved with straightedge and compass, and the constructibility of regular polygons.

At the end of World War I, in March 1919, a commission of the High Council for Public Education, presided by Luigi Credaro, and in which Pincherle participated as the representative of the mathematicians, examined the existing university regulations with a view to advancing proposals for their improvement. In his report about the degree program in mathematics, Pincherle addressed the problem of the dual aim of this degree – to direct young people into research, and to prepare mathematics teachers for secondary schools – repeating his proposal of 1903 that in the second biennium the degree program be differentiated between those who wished to obtain a scientific degree and those who instead intended to dedicate themselves to teaching. For those wishing to teach was to be instituted a "teaching of elementary mathematics of a critical, educational and methodological character, lasting two years, and with as much regard for geometrical and analytical aspects of mathematics, and as much for the foundations of the science of numbers as for that of magnitudes" (Nastasi 2002, pp. 82–84). Furthermore, the subjects of a more advanced character were to be kept to a minimum and treated in a different way:

> This is not meant to suggest that there is any conflict between elementary mathematics and advanced studies. The former are necessary to the latter, and without a course in higher analysis, and higher geometry, the necessary light cannot be shed on many points of the elements. But in these same courses of higher studies there is a different formulation that is appropriate for the two categories of candidates: for the future scientists a character that is eminently monographic is required; instead, one that is preferably propaedeutic for those who are directed to

teaching. (Nastasi 2002, p. 82)

Principal discussions at the Mathesis Congresses

Turin Congress (1898).	**L. Certo** (*Modifiche opportune nell'ordinamento degli studi matematici universitari*) proposes establishing university courses on the foundations of mathematics, history of mathematics and mathematical logic (PM 1899, 107–116).
Livorno Congress (1901).	**G. Pittarelli** (*Modificazioni da introdursi nell'insegnamento matematico superiore per la preparazione degli insegnanti secondari*) re-proposes the project presented previously by L. Cremona to create a Faculty of Philosophy that would unite Letters, Philosophy and the Sciences (ATTI, 137–164).
Preparatory assembly, Bologna (1903).	**S. Pincherle** proposes the institution, after the first two years of university, of a special school that would lead to a teaching degree, distinct from that in pure mathematics (BM 1903, 14).
Naples Congress (1903).	**G. Costanzi** (*Convenienza di rendere non obbligatoria la laurea in Matematica a chi vuol conseguire il Diploma di Magistero per le scuole medie*) points out the importance of elementary mathematics from an advanced standpoint in teacher training as well as the significance of applied mathematics (ATTI, 38–55).
Florence Congress (1908).	**C. Arzelà, G, Pittarelli, L. Certo** (*Training of mathematics teachers for middle schools*) **A. Padoa** (supported by G. Peano, G. Loria) formulates an item on the agenda expressing his disapproval of making a distinction between a scientific and a didactic degree in mathematics, and proposes establishing some chairs on history and foundation of mathematics (ATTI, 55).
Padua Congress (1909).	**G. Loria and A. Padoa** formulate an item on the agenda requesting the establishment of university chairs in mathematical methodology, and the introduction of a teaching practice in a secondary school compulsory for future teachers (ATTI, 51).
Genoa Congress (1912).	**G. Castelnuovo** invites the Assembly to formulate a proposal relative to the improvement of the organisation of the *Scuole di Magistero* and to the distinction between a scientific degree and a didactic degree (ATTI, 88).
Trieste Congress (1919)	**G. Voghera** (*Sulla preparazione degli insegnanti*). **S. Pincherle** places an item on the agenda proposing that a course in elementary mathematics from an advanced standpoint be established and also that the teaching of physics be strengthened at the university courses for future teachers (PM 1920, 62).
R.D. 8 August 1920 (B. Croce)	Abolition of the *Scuole di Magistero*.

Abbreviations

BM: *Il Bollettino di Matematica*

BUMPI: *Bollettino Ufficiale del Ministero della Pubblica Istruzione*

PM: *Periodico di Matematiche*

ATTI: *Atti del Secondo Congresso dei Professori di Matematica delle scuole secondarie*. Livorno: Giusti, 1902; *Atti del III Congresso fra i professori di matematica delle scuole medie italiane promosso dall'Associazione "Mathesis"*. Torino: Artigianelli, 1904; *Atti del I Congresso della Mathesis Società Italiana di Matematica.* Padova: Premiata Società Cooperativa, 1908; *Atti del II Congresso della Mathesis Società Italiana di Matematica*. Padova: Premiata Società Cooperativa, 1909; *Atti del III Congresso della Mathesis Società Italiana di Matematica.* Roma: Cooperativa Tipografica Manuzio, 1913.

References

Berzolari, L., & Bonola, R. (1909). Sopra una Enciclopedia di Matematiche Elementari. In *Atti del II° congresso della "Mathesis" Società Italiana di Matematica*, Allegato E, 1–5. Padova: Premiata Società Coperativa.

Enriques, F. (1907). Sulla preparazione degli insegnanti di Scienze. In *Quinto Congresso nazionale degli insegnanti delle scuole medie. Bologna, 25-26-27-28 settembre 1906, Atti*. Pistoia: Tip. Sinibuldiana, 69–78.

Gario, P. (2006). Quali corsi per la formazione del docente di matematica? L'opera di Klein e la sua influenza in Italia. *Bollettino della Unione Matematica Italiana. La Matematica nella società e nella cultura*, *8*, IX-A, 131–141.

Loria, G., & Padoa, A. (1909). Preparazione degli insegnanti di matematica per le scuole medie. *Atti del II° congresso della "Mathesis" Società Italiana di Matematica*, Allegato A: 1-10. Padova: Premiata Società Coperativa.

Nastasi, P. (2002). La Mathesis e il problema della formazione degli insegnanti. In Bolondi, G. (Ed.) 2002. La *Mathesis*. La prima metà del Novecento nella "Società Italiana di Scienze matematiche e fisiche." *Pristem/Storia* 5. Milano: Springer-Verlag Italia, 59–119.

Pincherle, S. (1906). Sulle riforme scolastiche da compiersi e in particolare su quelle relative all'insegnamento della matematica. *Il Bollettino di Matematica*, *5*(5–6), 83–87.

Appendix 2

The mathematics courses in the *Scuole di Magistero* in Italy[1]

No systematic study of the courses held by mathematicians in the *Scuole di Magistero* has yet been carried out, but archival research has brought some significant examples to light.

At the University of Turin the mathematics courses in the *Scuole di Magistero* were taught by various professors (Enrico D'Ovidio, Francesco Faà di Bruno, Francesco Siacci, Giuseppe Basso, Corrado Segre), but from 1891 they were taught only by Segre (1863-1924), the father of the Italian school of algebraic geometry. In addition to his courses on advanced geometry, for nineteen years Segre also ran a course for future teachers, from 1887–1888 to 1891–1892 and from 1907–1908 to 1920–1921.[2] His handwritten lesson notes – in particular *Lezioni di Geometria non euclidea* (1902–1903), *Vedute superiori sulla geometria elementare* (1916–1917), and [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*] (Giacardi 2002) – and archival documents clearly show the threefold approach of Segre's lessons: theory, methodology, and practice. He took up anew the themes of elementary mathematics studied at the secondary level, making evident from time to time the connections to higher mathematics; he also examined questions of methodology and didactics. Then, in the laboratories-classes, students were taught to impart clear lessons, documented and stimulating. In the notebook [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*], after beginning with some considerations on the nature of mathematics, the objectives of teaching, and the importance of intuition and rigor, Segre provides future teachers with some methodological instructions which are the fruit of his own teaching experience and of an attentive examination of legislative measures in various European countries and of educational issues debated at the time. Segre's principal points of reference were C. A. Laisant, E. Borel, J. Hadamard and H. Poincaré in France, and P. Treutlein, M. Simon, and Klein in Germany, mathematicians who were all involved in improving the role of intuition as opposed to a teaching too marked by logical rigor. In particular, he made Klein's

[1] By Livia Giacardi.
[2] From 1892-1893 to 1906-1907 he was substituted by Enrico d'Ovidio. See ASUT, *Conferenze della Scuola di Magistero di Scienze*, VII 84.

pedagogical assumptions his own (Giacardi 2010). From archival documents, in spite of gaps, it is possible to learn the number of men and women in the audience, and to note the progressive increase in the number of women.[3]

In Bologna the mathematics course at the *Scuole di Magistero* began in 1892-93 and was taught by Cesare Arzelà (1847-1912) up to 1898-1899. Documents in the archives show that Salvatore Pincherle (1853- 1936) taught the classes from 1899-1900 to 1920-21. Both Arzelà and Pincherle, besides being quite accomplished mathematicians in the field of analysis, were authors of well-received handbooks for secondary schools, and thus were particularly aware of problems of mathematics teaching. From the documentation we have, we can see that in 1893 Arzelà dealt with the following topics: "On maxima and minima that are determined by means of elementary algebra. General theory of magnitudes. Proportional magnitudes. The problems of elementary geometry (exposition of the book by Petersen). Negative numbers. Discussions about various textbooks.".[4] All of Pincherle's annual reports show the importance he attached to elementary mathematics from an advanced standpoint, and his growing interest in questions regarding the principles and foundations of mathematics. Also evident is an ever increasing attention in questions of methodology and didactics.[5] Perhaps it was a result of time spent with Federigo Enriques, who at that time was teaching in Bologna. Although he was not affiliated with the Scuole di Magistero of the University of Bologna, in 1916-1917 Enriques gave several lectures to future teachers on non-Euclidean geometry, which were collected and published in 1918 by Zanichelli under the title *Conferenze sulla Geometria non-euclidea*.

In Rome the lessons in mathematics in the Scuole di Magistero, from at least 1908-1909 until their suppression, were taught by Giulio Pittarelli (1852-1934), who was first a secondary school teacher and then professor at the university. From the register of his lessons,[6] we can see that his books of reference were the *Grundlagen der Geometrie* (1899) by David Hilbert, the *Questioni riguardanti le*

[3] In 1887-1888: 9 men; 1888-1889: 12 men, 1 woman; 1889-1890: 12 men, 2 women; 1890-1891: 12 men, 1 woman; 1906-1907: 3 men, 10 women; 1907-1908: 4 men, 13 women; 1921-1922: 2 men, 11 women. See *Quaderni 38*, in [Giacardi 2002] and ASUT, *Conferenze della Scuola di Magistero di Scienze*, VII 84.

[4] ASUB, *Scuole di Magistero* (pos. 53/b), busta 2 (1893-1908).

[5] ASUB, *Scuole di Magistero* (pos. 53/b), busta 3 (1880-1921). Reports show the following enrollments: in 1912-1913: 4 men and 12 women; 1914-1915: 7 men and 13 women; 1916-1917: 1 man and 19 women; 1917-1918, 19 women.

[6] ASUR, *Facoltà di Scienze. Libretti delle lezioni: Pittarelli Giulio.* The register of 1914-1915 shows 4 men and 25 women enrolled.

matematiche elementari by Enriques and the handbooks co-authored by Enriques and Ugo Amaldi. In any case, he was particularly sensitive to problems related to teacher training, as shown by his lengthy report, *Modificazioni da introdursi nell'insegnamento matematico superiore per la preparazione degl'insegnanti secondari*, presented during the congress of the Mathesis Association held in in Livorno in 1901. Here, after a historical survey and an overview of teacher training in Europe, he underlined how important it was that future teachers be cultured not only in science but in the humanities as well, and proposed making some courses in the faculty of letters and philosophy mandatory (Pittarelli 1902).

In addition to Pittarelli's courses, Guido Castelnuovo – an important member of the Italian school of algebraic geometry, a delegate to the ICMI, and, later, a member of its Central Committee and then vice president – also introduced a number of topics designed specifically for the training of future mathematics teachers into his geometry courses at the University of Rome. We can mention the following courses: *Geometria non-euclidea* (1910–1911), *Matematica di precisione e matematica di approssimazione* (1913–1914), *Indirizzi geometrici* (1915–1916), *Equazioni algebriche* (1918–1919), and *Geometria non-euclidea* (1919–1920) (Gario 2001-2003). In the introduction to the 1913–1914 course on the relationship between precise and approximate mathematics, Castelnuovo indicates the various ways in which future teachers can be trained, and expresses his preference for an "extensive education in the various areas of mathematics and in the sciences that have the greatest affinity with mathematics" because "this is more suited to broadening the horizons of the future teacher and putting the subject to be taught into its proper perspective". This makes it possible to shed ample light on the relationships between pure and applied mathematics, as well as to understand how mathematical concepts are formed beginning from observations of the outside world, and how mathematical results can in their turn occur in real life. As for this he quotes Klein, saying:

> The relationship between problems pertaining to pure mathematics and those pertaining to applied mathematics is very interesting and instructive. Klein, who dedicated a series of lectures to the subject (1901), describes the first of these as problems of "precise mathematics" and the second as problems of "approximate mathematics". In this course we will … more or less follow the general outline of Klein's course. Klein also had another reason for pursuing this line of enquiry, which was his desire to bridge the gap between mathematicians engaged in pure research and those who have to solve problems relating to applied mathematics (pp. 2–3).

In speaking of the mathematics courses in the *Scuole di Magistero* we cannot neglect to mention that of Giuseppe Veronese (1854-1917), founder of projective geometry of hyperspace from the synthetic point of view, and initiator of the study of non-Archimedean geometry. In fact, his celebrated treatise of 1891, *Fondamenti di geometria a più dimensioni e a più specie di unità rettilinee esposti in forma elementare*, is a collection of the lessons given in the *Scuole di Magistero* of the University of Padua.[7] Although the lengthy preface contains considerations of a didactic and methodological nature, and a historic study of the foundations of geometry is given in an appendix, the work is complex, contains original research, and the exposition is often obscure, rendering it unsuitable for a course for training secondary school teachers.

Abbreviations

ASUB: Archivio Storico dell'Università di Bologna

ASUR: Archivio Storico dell'Università di Roma

ASUT: Archivio Storico dell'Università di Torino

References

Gario, P. (2001–2003) (Ed.). *Guido Castelnuovo. Quaderni delle lezioni.* CD-ROM 1–6. Milan: Dipartimento di matematica.

Giacardi, L. (2002) (Ed.). *I Quaderni di Corrado Segre,* CD-ROM. Torino: Dipartimento di matematica, Università di Torino.

Giacardi, L.(2003a). Educare alla scoperta. Le lezioni di Corrado Segre alla Scuola di Magistero. *Bollettino dell'Unione Matematica Italiana. La Matematica nella società e nella cultura* (8), VI-A, 141–164.

Giacardi, L. (2003b). I manuali per l'insegnamento della geometria elementare in Italia fra Otto e Novecento. In Chiosso, G. (Ed.), *TESEO, Tipografi e editori scolastico-educativi dell'Ottocento* (XCVII-CXXIII). Milano: Bibliografica.

Giacardi, L. (2010). The Italian school of algebraic geometry and mathematics teaching in secondary schools. Methodological approaches, institutional and publishing initiatives. *International Journal for the History of Mathematics Education, 5*(1), 1-19.

Pittarelli, G. (1902). Modificazioni da introdursi nell'insegnamento matematico superiore per la preparazione degl'insegnanti secondari. *Atti del Secondo Congresso dei Professori di Matematica delle scuole secondarie*, Livorno: Giusti, 137-164.

Veronese, G. (1891). *Fondamenti di geometria a più dimensioni e a più specie di unità rettilinee esposti in forma elementare*, Padova: Tipografia del Seminario.

[7] For educational repercussions, see (Giacardi 2003b, pp. CX-CXIII).

Appendix 3

The courses in complementary mathematics[1]

It is possible to form an idea of the nature of the courses in complementary mathematics from a preliminary investigation of the programs carried out in some Italian universities, and from the various initiatives promoted by some mathematicians for teacher training. In 1922–1923 the course in complementary mathematics at the University of Rome was taught by Enriques, who also directed the teaching exercises; in the years immediately after, it was taught by Castelnuovo, with the teaching exercises directed by Alfredo Perna (1873–1965), a secondary school teacher who became a supervisor for the Ministry of Public Education, and then a non-tenured professor at the University of Rome.

In his course,[2] Enriques dealt with the foundations of geometry, non-Euclidean geometry by means of a historic approach, numeric sets, the Pythagorean theorem, the regular polyhedra, conics, and so forth, with brief mentions of the theory of relativity and a comparative criticism of textbooks for geometry. In the teaching exercises he dealt with proportions, triangles, the sphere, the developments of functions in series, continuous fractions, the geometry of the compass, the duplication of the cube, etc. With regard to this course he wrote to Gentile:

> … by means of those problems that are closer to elementary mathematics and which have a history that is twenty centuries old, we aim to reach young people with a vocation for teaching, who … must be protected from the risk of becoming mechanical propagators of a culture that they have received from outside and is truly foreign to their spirit" (Enriques to Gentile, Rome, 23 December 1922, in Guerraggio and Nastasi 1993, pp. 149–150).

Enriques was intensely involved in teacher training, first as president of the Mathesis Association beginning in 1919, and then as director of the *Periodico di Matematiche* beginning in 1921. In the letter to the readers that opened the 1921 issue of the *Periodico*, a journal expressly aimed at teachers, he presented an actual working program for the journal, which was at the same time a working program for teachers:

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[1] By Livia Giacardi.
[2] See the register of lessons and exercises in Giacardi (to appear: Appendices 1.2, 3).

teachers should study the science that they are teaching in depth from various points of view, so as to make evident the connections between elementary mathematics and higher mathematics; use the history of the science seeking to attain, not so much erudite knowledge, as a dynamic consideration of concepts and theories, through which students can recognise the unity of thought; and bring out the relationships between mathematics and the other sciences, and physics in particular, in order to offer a broader vision of science (Enriques 1921, pp. 3–4).

In his courses of complementary mathematics, Castelnuovo dealt with non-Euclidean geometry, again using a historic approach, problems that can and cannot be solved with straightedge and compass, the problem of cyclometry, continuous fractions, the theory of algebraic equations, the numbers *pi* and *e*, isoperimetric problems, maxima and minima of functions, and the calculus of variations. In the teaching exercises Perna went over some of the topics again, proposing exercises and "practical lessons" (probably taught by the students), and the carrying out of exercises assigned for the state examinations.[3]

In Turin, in the first year of its institution the course in complementary mathematics was taught by Segre,[4] who in all probability dealt with the same topics that he had taught in the course in the *Scuola di Magistero*. From 1925 to 1932 the course was taught by Peano, who had always been interested in teacher training. For about a decade, starting from 1915, he was the driving force behind a series of lectures called *Conferenze Matematiche Torinesi*. Many articles aimed at secondary school teachers, including a number written by women, developed out of these meetings and were published in the major journals for education. Peano's advice to future teachers was to seek rigor and simplicity, to use an economy of language, avoiding redundancies and vicious circles, and to take care to motivate and interest the students (Luciano and Roero 2008).

Abbreviations

ASUR: Archivio Storico dell'Università di Roma

ASUT: Archivio Storico dell'Università di Torino

[3] ASUR, *Facoltà di scienze, Libretti delle lezioni,1923–1926: Castelnuovo Guido, Perna Alfredo.*
[4] ASUT, Ufficio posta e protocolli, Università di Torino, *Fascicolo personale*.

References

Enriques, F. (1921). Ai lettori. *Periodico di Matematiche*, *4*(1), 1–5.

Gario, P. (2004). Guido Castelnuovo e il problema della formazione dei docenti di matematica. *Supplemento ai Rendiconti del Circolo Matematico di Palermo, s. 2, 74*, 103–121.

Giacardi, L. (to appear). Federigo Enriques (1871–1946) and the training of mathematics teachers in Italy. In Coen, S. (Ed.), *Mathematicians in Bologna 1861–1960.* Basel: Birkhäuser.

Guerraggio, A., & Nastasi, P. (1993). *Gentile e i matematici italiani. Lettere 1907–1943*. Torino: Bollati Boringhieri.

Luciano, E., & Roero, C. S. (2008). *Giuseppe Peano. Matematico e Maestro.* Torino: Dipartimento di Matematica, Università di Torino.

Appendix 4

Mathematics teachers appearing in the list of Italian mathematicians compiled by Tricomi[1]

Source: Tricomi, F. G. (1962). Matematici del primo secolo dello Stato unitario. *Memorie dell'Accademia delle Scienze di Torino, Classe Scienze Fisiche, Matematiche, Naturali, s.4, 1*, 1–120.

The work (Tricomi 1962) is a collection of short biographical notes about 371 people who died in the period 1 January 1861 to 31 December 1960 (first hundred years of unified Italy), who contributed in some way to the development of mathematical studies. In spite of some gaps and errors, inevitable in such a kind of work – the poor presence of women for one – Tricomi's collection is helpful for outlining the picture of an important period in the development of the Italian mathematical community. From Tricomi's collection we have extracted the following table, which lists those mathematicians who taught for about ten years in primary or secondary schools or for most of their professional life.

Note
We maintain the Italian names for the various kinds of schools, because there is not always a correspondence with other countries. Some information is added by the authors. Dates given are dd.mm.yyyy.

Surname, name, place and dates of birth and death	**Schools**	**Libera docenza (habilitation)**	**Institutional appointments in school**	**University appointments and career**	**Involvement in journals or collective works for teachers**	**Author of textbooks or didactic papers**	**Additional notes**
ALAGNA Rosario Partanna (Trapani) 12.7.1853 Palermo 1924	Middle schools in Palermo	Yes		Lecturer at Universities of Catania and Palermo			Charter member of *Circolo Matematico di Palermo*
ALASIA-DE QUESADA Cristoforo Sassari 21.11.1863/1869 Albenga (Savona) 19.11.1918	*Ginnasi* in various places, last in Albenga (Savona)				Founder and editor of the journal *Le Matematiche Pure e Applicate*	Didactic papers	Gold medal from the *Irish Academy* of Dublin (1911) for an article on astronomy
ALBEGGIANI Michele Luigi Palermo 7.3.1852 Palermo 16.11.1943	*Istituto Tecnico* in Palermo	Yes		Lecturer at University and *Scuola di Ingegneria* in Palermo			In 1904 president of *Circolo Matematico di Palermo*

[1] By Fulvia Furinghetti and Livia Giacardi.

AMALDI Italo Bozzolo (Mantova) 26.1.1864 Schio (Vicenza) 12.7.1940	*Istituti Tecnici*, last in Bologna		Principal of various *Istituti Tecnici*	Assistant at University of Pavia			
AMANZIO Domenico Marano (Naples) 2.2.1854 Naples 19.8.1908	*Scuola Militare* and *Istituto Tecnico* in Naples	Yes				Textbooks	Assistant at *Osservatorio di Capodimonte*
AMODEO Federico Avellino 8.10.1859 Naples 3.11.1946	*Istituto Tecnico* in Naples			Lecturer at University of Naples		Didactic papers	Member of *Accademia Pontaniana*
ASCOLI Guido Leghorn 12.12.1887 Turin 10.5.1957	Secondary schools in various places, last in Turin			Full professor at Universities of Pisa, Milan, and Turin		Didactic papers	Correspondent of *Accademia Nazionale dei Lincei* and of *Accademia delle Scienze di Torino*. Organizer of courses for teacher training. President of Piedmont chapter of *Mathesis* Association and of *Commissione Italiana per l'Insegnamento della Matematica*
BARDELLI Giuseppe Sedriano (Milan) 8.4.1837 Milan 1.3.1908			Principal of *Istituto Tecnico* in Milan	Lecturer at *Politecnico* of Milan			Member of *Istituto Lombardo*. Mayor of Sedriano and member of Milan city council
BARONI Ettore Pisa 16.2.1866 Rome 26.10.1918	Various schools, last *Liceo* in Rome			Assistant at University of Pisa	Contributor to *Collectanea* edited by F. Enriques		
BELLACCHI Giacomo Altamura (Bari) 9.12.1838 Florence 25.2.1924	*Scuola Militare* and *Istituto Tecnico* in Florence					Textbooks, also at university level. Didactic papers	He had no university degree, only a Diploma in *Scuola Normale Superiore* of Pisa

BENEDETTI Piero Castel del Piano (Grosseto) 17.10.1876 Pisa 16.3.1933	*Istituto Tecnico* in Pisa		Member of *Consiglio Superiore della Pubblica Istruzione*	Lecturer at University of Pisa	Contributor to *Enciclopedia delle matematiche elementari*	Textbooks (with C. Rosati). Didactic papers	President of *Federazione Nazionale degli Insegnanti Medi*
BESSO Davide Trieste 28.7.1845 Frascati (Rome) 8.8.1906	Various schools, last *Istituto Tecnico* in Rome			Full professor at University of Modena	Founder and editor of the journal *Periodico di Matematica*	Didactic papers	
BETTAZZI Rodolfo Florence 14.11.1861 Turin 26.1.1941	Various schools, last *Liceo* in Turin			Assistant at University of Pisa. Lecturer at *Accademia Militare* in Turin	With A. Lugli and F. Giudice founder of *Mathesis* Association. First president of *Mathesis*	Didactic papers	
BETTINI Bettino Filottrano (Ancona) 5.1.1860 Osimo (Ancona) 5.2.1930	*Liceo* in Osimo					Textbooks. Didactic papers	
BONACINI Carlo Modena 15.8.1867 Modena 1.1.1944	Various schools, last *Liceo* and *Istituto Tecnico* in Modena			Lecturer at University of Modena			Director of the Geophysical Observatory of Modena. Pioneer of colored photos and stereophony
BONOLA Roberto Bologna 14.11.1874 Bologna 16.5.1911	Various schools			Lecturer at University of Pavia		Didactic papers	
BRAMBILLA Alberto S. Zenone Po (Pavia) 13.6.1857/1858 Naples 3.12.1908	Various schools, last *liceo* in Naples	Yes		Lecturer at University of Naples		Didactic papers	
BRUSOTTI Luigi Pavia 11.9.1877 Padua 30.4.1959	Various schools, last *Istituto Tecnico* in Pavia			Assistant at University of Pavia. Full professor at Universities of Cagliari, Pisa and Pavia	Contributor to *Enciclopedia delle matematiche elementari*	Didactic papers	Member of *Istituto Lombardo*. Corresponding member of *Accademia Nazionale dei Lincei*

BURALI-FORTI Cesare Arezzo 13.8.1861 Turin 21.1.1931	Various schools in Turin			Lecturer at *Accademia Militare* in Turin		Didactic papers	Famous for the "Burali-Forti paradox"
CALÒ Benedetto Bagno a Ripoli (Florence) 22.11.1869 Bagnoli (Naples) 23.8.1917	Various schools, last *Istituto Tecnico* in Naples	Yes		Assistant at University of Turin	Contributor to *Collectanea* edited by F. Enriques	Didactic papers	
CANDIDO Giacomo Guagnano (Lecce) 10.7.1871 Galatina (Lecce) 30.12.1941	Various schools		In 1927 appointed as an organizer of the new *Liceo* in Brindisi, where later he was the principal until 1936		Founder and editor of the journal *La Matematica Elementare*	Didactic papers	
CASSANI Pietro Venice 4.6.1832 Venice 6.6.1905	Various schools, *Istituto Tecnico* in Venice					Textbooks. Didactic papers	Member of *Istituto Veneto* of Venice
CATTANEO Paolo Padua 20.4.1878 Padua 13.1.1960	Various schools, *Istituto Nautico* in Venice					University textbooks. Didactic papers	
CAZZANIGA Tito Camillo Virgilio (Mantua) 9.4.1872 Virgilio (Mantua) 30.10.1900	Various schools					Didactic papers	Sentenced to die for political reasons
CHINI Mineo Massa 8.5.1866 Florence 11.11.1933	Various schools		Principal of *Istituto Tecnico* in Florence	Lecturer at University of Florence		Textbooks. Didactic papers	
CIAMBERLINI Corrado Cingoli (Macerata) 1.5.1861 Fermo (Ascoli Piceno) 2.11.1944	Various schools, last *Liceo* in Fermo			Assistant at University of Rome		Textbooks. Didactic papers	Nominated *Commendatore* in the Order of the Crown of Italy

CIPOLLA Michele Palermo 28.10.1880 Palermo 7.9.1947	Various schools			Full professor at Universities of Catania and Palermo		Textbooks (with V. Amato and G. Mignosi). University notes for prospective teachers. Didactic papers	Doctorate degree *honoris causa* from University of Sofia. Member of *Accademia Nazionale dei Lincei* and other Italian Academies
CODAZZI Delfino Lodi (Milan) 7.3.1824 Pavia 21.7.1873	Various schools			Full professor at University of Pavia			Honorable mention by *Académie française*
CONTI Alberto Florence 3.12.1873 Florence 18.10.1940	Various schools, last in Florence				Founder and editor of the journals *Il Bollettino di Matematiche e di Scienze Fisiche e Naturali* and *Bollettino di Matematica*	Textbooks. Didactic papers	
DAINELLI Ugo Tuscany 1.3.1849 Rome 19.12.1906	*Istituti Tecnici* in Palermo, Como and Rome			Assistant at University of Bologna			Specialization in Paris and Dresden
DELL'AGNOLA Carlo Alberto Taibon (Belluno) 23.6.1871 Venice 15.8.1956	*Istituti Tecnici* in Cagliari and Venice	Yes		Assistant and lecturer at University of Padua. Full professor and rector at University of Venice (Cà Foscari)		Didactic papers	
DEL PRETE Guelfo Lucca 1873 Lucca 25.10.1901	*Scuole normali* in Avellino and Naples					Textbooks (with G. Garbieri)	
D'ESCAMARD Vincenzo S. Felice a Cancello (Naples) 27.4.1874 Naples 16.9.1941	Various schools, last in Naples						
DE ZOLT Antonio Conegliano (Treviso) 8.8.1847 Milan 1926	*Liceo* in Milan						Famous for the "De Zolt postulate"

DUCCI Enrico Fermo (Ascoli Piceno) 15.5.1864 Naples 29.7.1940	Various schools, *Collegio Militare* in Naples					Didactic papers	
EUGENIO Vito 19.8.1850 Naples 25.2.1907	*Istituto Tecnico* in Naples		Principal of *Istituto Tecnico* in Naples			Didactic papers	
FAIFOFER Aureliano Borgo Valsugana (Trento) 4.8.1843 Venice 1.3.1909	*Liceo* in Venice			Assistant at University of Padua		Textbooks. Didactic papers	
FAZZARI Gaetano Tropea (Catanzaro) 6.10.1856 Messina 13.7.1935	Various schools, last *Liceo* in Palermo				Founder and editor of the journal *Il Pitagora* for secondary students	Didactic papers	
FINZI Aldo Mantua 1878 Rome November 1934	Various schools		Principal, regional inspector, provincial director of Education in Campania		Contributor to *Enciclopedia delle matematiche elementari*	Didactic papers	
FORTI Angelo Pesaro 1818 Rome 23.12,1900	*Scuole Tecniche* and *Liceo* in Pisa			He was offered positions in university		University textbooks. Numerical tables of hyperbolic functions	
FRATTINI Giovanni Rome 8.1.1852 Rome 21.7. 1925	*Liceo*, *Istituto Tecnico*, *Collegio Militare* in Rome			He was offered positions in university		Textbooks. Didactic papers	Famous for his studies on group theory
FUORTES Tarquinio Castrigano del Capo (Lecce) 29.7.1848 Latiano (Brindisi) 30.4. 1927	*Collegio Militare* and I*stituto di Belle Arti* in Naples					Didactic papers	
GALLUCCI Generoso Naples 12.9.1874 Naples 17.2.1941	Various schools, last *Liceo Artistico* in Naples		Regional inspector	Lecturer at University of Naples		Didactic papers	Member of *Accademia delle Scienze* in Naples and *Accademia Pontaniana*

GAMBIOLI Dionisio Pergola (Pesaro) 11.9.1858 Rome 4.11. 1941	Various schools, last *Istituto Tecnico* in Rome					Didactic papers	Translator of the book of history of mathematics by W. Rouse Ball. Editor with V. Volterra and G. Loria of G. Fagnano's works
GARBIERI Giovanni Bologna 14.9.1849 Genoa 6.2.1931	Primary school in Bologna, later various secondary schools		Principal of *Istituto Nautico* in Savona	Lecturer at University of Padua. Full professor at University of Genoa		Textbooks. Didactic papers	Active in the *Scuola di Magistero* in Genoa
GAZZANIGA Paolo Soresina (Cremona) 26.7.1853 Venice 18.10.1930	*Liceo* in Padua			Lecturer at University of Padua		Textbooks (with G. Veronese)	Specialization in Germany
GHERARDELLI Giuseppe Florence 1.1.1894 Florence 1.7.1944	*Licei* in Turin and Florence			Assistant at University of Turin. Full professor at University of Pavia			
GIGLI Duilio Sansepolcro (Arezzo) 8.1.1878 Pavia 10.5.1933	Various schools, last *Liceo* in Pavia			Lecturer at University of Pavia	Coeditor of *Enciclopedia delle matematiche elementari*		
GIUDICE Francesco Codevilla (Pavia) 1.3.1855 Voghera (Pavia) 11.8.1936	Various schools, *Istituti Tecnici* in Genoa and Pavia	Yes		Lecturer at University of Pavia	With A. Lugli and R. Bettazzi founder of *Mathesis* Association	Textbooks. Didactic papers	
GIULIANI Giulio Pisa 6.3.1859 Barga (Lucca) 1940	Various schools, *Liceo* in Pisa					Textbooks. Didactic papers	
GIULOTTO Virgilio Mantova 5.5.1877 Milano 18.4.1945	Various schools						Winner of Ministry award for mathematical research
GRAMEGNA Maria Tortona (Alessandria) 11.5.1887 Avezzano (L'Aquila) 13.1.1915	*Scuola Normale* in Avezzano						

LENZI Enrico Taormina (Messina) 10.5.1880 Genoa 19.9.1945	Various schools, last *Liceo* in Turin			Assistant at University of Naples. Full professor at University of Genoa			
LICOPOLI Guglielmo Naples 3.7.1883 Naples August 1913	Various schools			Lecturer at University of Naples			
LO MONACO-APRILE Luigi Palermo 4.4.1875 Turin 1915	Various schools			Assistant at University of Turin			
LUGLI Aurelio Modena 6.12.1853 Rome 27.5.1896	Various schools, last *Istituto Tecnico* in Rome		Assistant at the Central Meteorological Office		With R. Bettazzi and F. Giudice founder of *Mathesis* Association. Editor of the journal *Periodico di Matematica*	Didactic papers	
MACCAFERRI Eugenio Massa Lombarda (Ravenna) 22.10.1870 Bologna 22.7.1953	Various schools, last *Istituto Tecnico* in Piacenza		Principal of various schools	Assistant at University of Bologna		Textbooks. Didactic papers	
MARLETTA Giuseppe Catania 10.10.1878 Catania 20.3.1944	Various schools	Yes		Assistant, lecturer, full professor at University of Catania		Didactic papers	Member and president of *Accademia Gioenia*
MEDICI Siro Santafiora (Siena) 12.4.1883 In the battle of Isonzo 22.10.1917	Various schools, last *Istituto Tecnico* in Florence			Assistant at University of Pisa		Didactic papers	
MERCOGLIANO Domenico Mercogliano (Avellino) 23.6.1873 Naples 6.3.1936	Various schools, last *Istituto Magistrale* in Naples					Italian translation of J. W. Young, *Lectures on fundamental concepts of algebra and geometry*	In 1935 Ministry award of *Accademia Nazionale dei Lincei*

MIGNOSI Gaspare Palermo 5.1.1875 Palermo 11.6.1951	Various schools, last *Liceo Scientifico* in Palermo			Assistant at University of Palermo. Full professor at Universities of Cagliari and Palermo		Textbooks (with M. Cipolla). Didactic papers	
MURER Vittorio Como 12.7.1860 Turin 10.8.1909	Various schools, last *Liceo* in Turin					Didactic papers	Member of *Circolo Matematico di Palermo*
NOBILE ARMINIO Naples 12.8.1838 Naples 14.6.1897	*Istituto Tecnico* in Naples			Full professor at University of Naples			Member of *Accademia Nazionale dei Lincei* and *Accademia delle Scienze* in Naples
NOVI Giovanni Naples 2.1.1827 Pisa 10.12.1866	*Liceo Militare* in Pisa			Full professor at University of Pisa		Translator of textbooks	
ORTU CARBONI Salvatore Sassari 7.12.1859 Genoa 6.11.1939	Various schools			Full professor (later director) at the *Scuola superiore di Commercio* at University of Genoa		Didactic papers	
PADOA Alessandro Venice 14.10.1868 Genoa 25.11.1937	Various schools, last *Istituto Tecnico* in Genoa	Yes		Lecturer at University of Genoa	Contributor to *Collectanea* edited by F. Enriques	Didactic papers	Contributor to International Congresses of Mathematicians
PALAMÀ Giuseppe Sogliano Cavour (Lecce) 22.5.1898 Sogliano Cavour (Lecce) 14.10.1959	*Liceo* in Lecce			He was offered positions in university		Didactic papers	Winner of various awards (*Accademia Nazionale dei Lincei*, *Mathesis*, etc.)
PALATINI Francesco Bassano del Grappa (Vicenza) 25.7.1865 Castell'Azzara (Grosseto) 12.6.1940	Various schools, last *Istituto Tecnico* in Turin					Didactic papers	
PANNELLI Marino Macerata 16.11.1855 Macerata 16.4.1934	*Istituto Tecnico* in Rome	Yes		Lecturer at University of Rome			

PENNACCHIETTI Giovanni Arcevia (Ancona) 25.7.1850 Rome 21.8.1916	Various schools			Full professor at University of Catania			
PEPOLI Alessandro Palermo 15.3.1852 Palermo 22.12.1930	Various schools in Palermo						Librarian of *Circolo Matematico di Palermo*
PICCIATI Giuseppo Piombino (Livorno) 30.10.1868 Venice 11.3.1908	Various schools, last *Scuola di Macchinisti di Marina* in Venice						
PINTO Luigi Castellana (Bari) 6.5.1846 Castellana (Bari) 27.2.1920	Various schools			Full professor at University of Naples. Rector of this University			Member of *Accademia delle Scienze* in Naples and of *Accademia Pontaniana*
PIRONDINI Geminiano Modena 3.10.1857 Rome 18.12.1914	Various schools, last *Istituto Tecnico* in Rome					Didactic papers	
PITTARELLI Giulio Campochiaro 3.2.1852 Roma 2.3.1934	*Istituto Tecnico* in Chieti, *Liceo* in L'Aquila and Rome			Full professor at University of Rome			
PITTEI Costantino Prato (Florence) 8.2.1839 Florence 5.6.1912	Various schools, last *Istituto Tecnico* in Florence						
PREDELLA Pilo Mantua 24.12.1863 Turin 4.1.1939	Various schools, last in Turin						
RAZZABONI Amilcare Modena 3.3.1855 Bologna 15.11.1920	Various school, last *Liceo* in Bologna			Assistant at University of Pisa			
RETALI Virginio Marciana Marina (Livorno) 24.11.1853 Milan 5.5.1930	Various schools, last *Liceo* in Milan				Contributor to *Enciclopedia delle Matematiche Elementari*	Didactic papers	Member of *Circolo Matematico di Palermo.* Ministry award by *Accademia Nazionale dei Lincei*

RIBONI Gaetano 15.8.1854 24.2.1932	Various *Istituti Tecnici*, last in Milan					Textbooks. Didactic papers	President of Milan chapter of *Mathesis* Association
RICCÒ Annibale Modena 14.9.184 Rome 23.9.1919	*Istituto Tecnico* in Modena			Assistant at University of Modena. Full professor at Universities of Palermo, Catania, and *Scuola di Ingegneria* in Naples. Rector of University of Catania			Member of *Accademia Nazionale dei Lincei* and of *Accademia Gioenia.* Vice-president of *International Astronomical Union*
RINDI Scipione Lucca 29.10.1859 Montecatini (Pistoia) 23.2.1952	Various schools, last *Istituto Tecnico* in Lucca					Didactic papers	
ROSATI Carlo Livorno 24.4.1876 Pisa 19.8.1929	Various schools, last *Istituto Tecnico* in Pisa			Assistant, full professor at University of Pisa		Textbooks (with P. Benedetti). Didactic papers	
RUFINI Enrico Rocca di Papa (Rome) 26.11.1890 Rome 3.11.1924	Various schools, last *Liceo* in Rome					Didactic papers	Member of the *R. Commissione Vinciana*. Editor of Archimedes's *Method*
SALVATORE DINO Nicola Torre Annunziata (Naples) 12.11.1843 Portici (Naples) 2.21.1919	Various schools, last *Liceo* in Naples			Full professor at Universities of Rome and Naples			Member of *Accademia delle Scienze* of Naples and *Accademia Pontaniana*
SBRANA Umberto Pisa 2.3.1882 Genoa 17.11.1942	Various schools, last *Istituto Tecnico* in Genoa			Assistant at University of Pisa			Position of specialization Lavagna. Librarian at University of Genoa
SCARPIS Umberto Padua 1861 Bologna 27.12 1921	Various schools, last *Liceo* in Bologna	Yes		Lecturer at University of Bologna	Contributor to *Collectanea* edited by F. Enriques. Coeditor of *Bollettino di Matematica*	Didactic papers	

SCORZA Gaetano Morano Calabro (Cosenza) 29.9.1876 Rome 6.8.1939	Various schools	Yes	Member of *Consiglio Superiore della Pubblica Istruzione*	Assistant at Universities of Turin and Pisa. Full professor of at Universities of Cagliari, Parma, Catania, Naples, and Rome		Didactic papers	Member of *Accademia Nazionale dei Lincei*. Senator of the Kingdom of Italy
SENIGAGLIA Ermanno 16.5.1889 died in war 31.5.1916	Various schools, last in Venice					Didactic papers. Unfinished book on methodology	
SIBIRANI Filippo S. Agata Bolognese (Bologna) 4.2.1880 Bologna 9.3.1957	*Istituto Tecnico* in Milan		In 1945-46 as a prefectorial commissar reorganized the Italian Mathematical Union	Assistant at University of Bologna. Lecturer at Universities of Parma, Bologna, and Pavia. Full professor at Universities of Trieste and Pavia. Rector of new Institute of Economical and Trade sciences at the University of Bologna	Contributor to *Enciclopedia delle matematiche elementari* and to *Repertorio …* edited by M. Villa	Didactic papers	Member of *Accademia delle Scienze* in Bologna and of *Accademia Gioenia*
SITTIGNANI Maria Giovanna Ravenna 10.10.1879 Genoa 4.8.1947	Various schools, last *Liceo* in Genoa					Didactic papers	Contributor to an International Congress of Mathematicians
TENCA Luigi Gambara (Brescia) 8.9.1877 Florence 26.8.1960	Various schools		Principal of a school. Provincial director of education in Bergamo and Pistoia	Assistant at University of Pavia		Editor of the journal *Il Bollettino di Matematiche e di Scienze Fisiche e Naturali.* Didactic papers	Was a general during WWI
TEOFILATO Pietro Naples 28.8.1879 Rome 31.8.1952	Various schools, last *Collegio Militare* in Rome			Full professor at Universities of Cagliari, Parma, Rome			Member of *Accademia Pontificia dei Nuovi Lincei*

TIMPANARO Sebastiano Tortorici (Messina) 2.1.1888 Pisa 22.12.1949	*Licei*			Assistant at University of Parma			Director of *Domus galileiana* in Pisa. Secretary of the Italian Group of History of Science
TORELLI Gabriele Naples 29.3.1849 Naples 7.11.1931	Various schools, last *Istituto Tecnico* in Naples			Assistant and lecturer at University of Naples. Full professor at Universities of Palermo and Naples			Member of *Accademia delle Scienze* of Naples and *Accademia Pontaniana*
VAILATI Giovanni Crema (Cremona) 23.4.1863 Rome 14.5.1909	Various schools		Member of a royal commission for reforming Italian secondary school	Assistant at University of Turin		Didactic papers	
VASSURA Giuseppe Faenza (Ravenna) 1.3.1866 Rome 24.1.1949	Various schools						Editor of E. Torricelli's works (with G. Loria)
VENERONI Emilio Milan 5.11.1874 18.6.1927	*Istituto Tecnico* in Pavia	Yes	Inspector of secondary schools	Lecturer at University of Pavia		Didactic papers	
VENTURI Adolfo Florence 22.9.1852 Palermo 28.12.1914	*Liceo* in Como			Full professor and rector at University of Palermo			Winner of Royal Prize for Astronomy
VERGERIO Attilio Valdobbiadene (Treviso) 19.1.1877 Naples 24.12.1937	Various schools	Yes		Assistant at University of Parma. Lecturer at University of Bologna		Didactic papers	
VITALI Giuseppe Ravenna 26.8.1875 Bologna 29.2.1932	Various schools, last *Liceo* in Genoa			Lecturer at University of Genoa. Assistant at University of Pisa. Full professor at Universities of Modena, Padua Bologna		Didactic papers	Member of *Accademia Nazionale dei Lincei* and of *Accademia* in Bologna

VITI Rodolfo 11.9.1874 Bologna 8.2.1929	*Istituto Tecnico* and *Liceo Scientifico* in Bologna					Textbooks	
VOLPICELLI Paolo Rome 8.1.1804 Rome 14.4. 1879	Various secondary schools in Rome			Full professor at University of Rome			Member and secretary of *Accademia Nazionale dei Lincei*

Appendix 5

Female teachers[1]

The community of teachers who contributed to the development of mathematical education in Italy also included women quite early on. In spite of the limits imposed by law and by prejudices in society and among colleagues, women slowly became the backbone of Italian education, first in primary schools and later also in secondary schools. At the turn of the century the profession of teaching was one of those most accessible for women (both married and single).

With regard to mathematics, Ravà (1902) reports that the first woman graduated in mathematics in 1887 and that by 1900 there were 20 women graduates in this discipline. Some women also attempted mathematical research (Fenaroli et al. 1990), but it would not be until 1921 that a woman achieved a chair in mathematics. The majority of those who graduated in mathematics became teachers in schools. Some of them showed a genuine and active commitment to their profession by contributing to didactic journals, participating in meetings of the Mathesis Association, and attending International Congresses of Mathematicians (Furinghetti 2002; Furinghetti 2008). Of the three women mentioned by Tricomi (1962), two were mathematics teachers.

As happened in the case of men, professional commitment often was accompanied by social commitment. Just to give an example: two women with *laurea* in mathematics, Lia Predella Longhi and Beatrice Sacchi, were among the 26 women who signed the petition for female suffrage published in the journal *Vita* on March 11, 1906 (Babini and Lama 2000, p. 170). Predella Longhi (1870 – after 1939) was a mathematics teacher for many years and wrote papers on mathematics and on mathematics teaching.

References

Babini, V. P., & Lama, L. (2000). *Una donna nuova. Il femminismo scientifico di Maria Montessori*. Milano: F. Angeli.

[1] By Fulvia Furinghetti.

Fenaroli, G., Furinghetti, F., Garibaldi, A. C., & Somaglia, A. M. (1990). Women and mathematical research in Italy during the period 1887–1946. In L. Burton (Ed.), *Gender and mathematics. An international perspective* (pp. 144–155). London: Cassell.

Furinghetti, F. (2002). Il *Bollettino della Mathesis* dal 1909 al 1920: pulsioni tra temi didattici internazionali e nazionali. *PRISTEM/Storia. Note di matematica, Storia, Cultura, 5*, 31–58.

Furinghetti, F. (2008). The emergence of women on the international stage of mathematics education. *ZDM – The International Journal on Mathematics Education*, *40*, 529–543.

Ravà, V. (1902). *Le laureate in Italia. Notizie statistiche*. Roma: Cecchini.

Tricomi, F. G. (1962). Matematici del primo secolo dello Stato unitario. *Memorie dell'Accademia delle Scienze di Torino, Classe Scienze Fisiche, Matematiche, Naturali, s. 4, 1*, 1–120.